MAURITIUS RESEARCH COUNCIL

INNOVATION FOR TECHNOLOGY

INVESTIGATING THE COMMON CORE CONSTRUCTS IN STUDENTS ACQUISITION OF LOGICO-MATHEMATICAL CONCEPTS IN PHYSICS AT HSC LEVEL

Final Report

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MAURITIUS RESEARCH COUNCIL

Address:
La Maison de Carné
Royal Road
Rose Hill
Mauritius

Telephone:	(230) 465 1235
Fax:	(230) 465 1239
e-mail:	mrc@intnet.mu
Website:	www.mrc.org.mu

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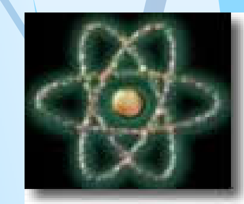

Investigating the common core constructs in students acquisition of logico-mathematical concepts in physics at HSC level

Prof V K P PARMESSUR *(Principal Investigator)*
Dr Y RAMMA
Mrs A N RAMDINNY
Mr H. BESSOONDYAL

TABLE OF CONTENTS

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Investigating the common core constructs in student's acquisition of logico-mathematical concepts in physics at HSC level.

Prof. V K P Parmessur [Principal Investigator]
Dr Yashwantrao Ramma
Dr Anita N Ramdinny
Hemant Bessoondyal

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Introduction

The Research Study

It is a well established fact that students encounter difficulty in learning physics at the secondary level of education. Moreover, it is not a surprise that physics is considered as one of the most difficult subject in our secondary school curriculum and this might be one of the main causes of students' under representation in the subject at the Higher School Certificate level.

Researches have shown that there are numerous factors associated with students' choice of subjects at the upper secondary level of education, for example, out-of-school factors, biological differences, teaching/learning strategies, role models, etc...

The teaching/ learning process, in our view, plays a very important role in encouraging or dissuading students from opting for physics as a core subject in higher classes. Physics educators should not be seen to form part of the dissuading factor and in this perspective our research team has developed a new teaching strategy appropriate to both girls and boys where individual difference is catered for.

There is a growing need for appropriate actions to be taken in the field of pedagogy and curricular activities in the light of growing concern at policy level to improve the quality of education in our schools. Most of the current and future developments in our society depend largely on the degree of scientific and technological impetus of the nation. Every single educator should form part of the driving force that would bring about a paradigm shift in our educational policy.

The aim of this research study was to:

(i) determine whether the teaching of physics is carried out comprehensively using appropriate mathematical concepts,

(ii) find out whether students encounter conceptual difficulty in learning physics,

(iii) investigate whether there is awareness of logico-mathematical concepts in physics from (a) the teachers', and (b) the students' point of view.

(iv) Investigate whether there are difficulties in the teaching of physics (from the teachers' point of view),

(v) Find out whether students encounter difficulty in logic related to problem solving in physics.

This consisted in:

(i) analysing the mathematics and physics curriculum at secondary level with a view to:
   1. identifying the common core constructs,
   2. establishing their degree of interdependence.

(ii) Observing physics classes to find out whether there is evidence of using common core constructs in the teaching of physics,

(iii) Conduction of unstructured interviews with physics teachers to investigate the problems faced by students,

(iv) Conduction of unstructured interviews with students,

(v) Designing and administering questionnaires to teachers and students,

(vi) Conduction of physics classes using core constructs,

(vii) Evaluating the lessons.

Chapter 1

Literature Review

Introduction

Mathematics is a language which is formed in the human mind whereas physics is the fundamental study of nature. It is a common fact that mathematics forms the basis of physics and it also a known fact that physics is considered as one of the most difficult subject in the school curriculum. Physics knowledge comprises of "mathematical and conceptual aspects" (Roth & Roychoudhury, 2003, pp S127); many students find it difficult to bridge a link between the two notions. It is not a surprise that a considerable number of students in the science stream will prefer to opt for chemistry or biology rather than physics.

At the secondary level of education, students studying physics usually experience difficulty in developing conceptual knowledge as well as in problem solving in order to solve hierarchical problems in physics. It is appropriate to point out that students' use of mathematics in physics mean that any time students invoke ideas from mathematics, such as equations or graphs for their conceptual development.

Review of Related Literature

Physics provides us with knowledge about the world in which we live. In order to understand physics it is necessary to:

1. develop a qualitative understanding of physical phenomena on Earth and in the Universe,
2. develop analytical skills with a view to quantifying the qualitative understanding,
3. be able to support acquired knowledge with corresponding facts.
4. develop a critical mind and problem solving skills.

One of the major aspects of physics is to be able to think critically and be able to analyse problems systematically. The qualitative understanding of physical phenomena requires the ability to apply logic to interpret various possibilities of reasoning and the accompanying thought processes. Being able to apply logic in some circumstances is insufficient to understand physics without a firm grasp of facts and figures. Data from historical events clearly stipulate that most major discoveries have been initiated by a prediction within a specific paradigm (Kuhn, 1970). Prediction on its own does not lead to scientific discoveries unless considerable effort is made to ascertain its existence (at least theoretically). Observation, investigation, experimentation and reporting consist of the four major elements of scientific process of thinking phases. At the initial stage, observation which incorporates any of the five senses, leads to investigation. Investigation raises a series of questions that needs to be systematically examined with verifiable answers. A thorough search for consistent answers implies structured methodology of inquiry and observable experimentation procedures. Finally, results stimulate further interest. Physics involves all the above stages of inquiry. The ability to self-direct one's thinking during these four stages leads to logical and organised critical thinking and, in turn, the development of rational mind may be achieved.

Many studies (e.g., Sommers, 1986; Blin-Stoyle, 1993, Cronin & Roger, 1999) confirm the low representation of students in physics. Students prefer to opt for chemistry and biology rather than physics; this may imply that they have already built up a negative attitude towards physics. Science, in particular is believed to consist of mystifying beliefs (Harding, 1991) due to the abstraction of a number of concepts which are remote from real life situations. These beliefs are deep rooted in the minds of the students and they act as a cognitive barrier to understanding of physics concepts. The abstractness of some physics concepts added with the '*chalk and talk*' mode of delivery of lessons leave no choice for students, but to shy away from physics. Branson *et al.* (1998) reveal that:

> Physics courses have not been designed to introduce students to the physics community as it really is, to show how physics links with human culture, or to reveal how new ideas in science develop (p. 25)

Moreover, students tend to accept information from teachers without or with little assimilation and eventually to memorise it irrespective of whether they have understood it or not (Din-Yan, 1998). It is the duty of the teachers to invoke students into critical and rational thinking as propounded by Parmessur et al (2003) whereby the students are led towards a "logically sequenced and organized summary" (p. 101) of the problem solving activity. Such a procedural activity enables the learners to continuously access and verify their own cognitive levels of performance, thus engaging them in their own process of learning. Ramma (2001) clearly showed that some students have a negative attitude towards physics, as propounded by one student:

> "every teacher claims that physics is a difficult subject and this makes it harder to us. Physics is too a vast subject and not all the subject matter is discussed in class + there is too much spoon-feeding and during the exams-we cannot deal with difficult numbers as we cannot think for ourselves. Teachers are too much syllabus-oriented."

There is a need to point out that "all teaching is to be carried out artistically" (Prange, 1996, p. 57) and that the act of educating must at no time happen at random.

There is truly a mismatch between what is taught in school and what goes on in the real world. Fisher (2001) rightly points out that "in school physics we too often rob students of the full range of such comparisons [of models] by choosing over-constrained situations" (pp. 75). The teaching of physics has merely changed; the traditional lecture type still predominates (Ramma, 2001).

It is unfortunate to contemplate that despite the fact that a large number of researches have been conducted in conjunction with good practice in teaching and learning of physics, the overall quality of the acquisition of worthwhile knowledge in physics is still unsatisfactory, irrespective of the number of low enrolment of students in physics as a school subject at the secondary level of education. Trumbull (1999) rightly views learning as "an active process that requires learners to grapple with the content until they develop their own understandings that will be long-lasting and useful" (p. 2).

There is a serious disparity between students' and teachers' notion of science (Osborne et al., 1998). Students would link science to technology and personal computers, while teachers would relate it to a "series of milestones" (p. 29) represented by a number of discoveries. Such a disparity impedes a logical flow of ideas between teachers and students.

A number of studies (for e.g., Driver, 1985; Wellington, 1994) have shown that science lessons tend to be basically focused on the transmission of a body of knowledge, rather than providing the opportunities for learners to develop skills pertaining to the development of "knowledge structures" (Mintzes et al., 1999, p. 10). Students who learn concepts by rote do not develop these structures and they inevitably produce alternative conception (misconception) in order to explain an event or a physical phenomenon. The gradual building up of misconceptions leads to a mismatch between conceptually correct and incorrect items which inevitably may create a state of confusion in the mind of the learner. Students might develop negative attitudes towards the subject. Osborne et al. (1998) points out that attitudes of students, in particular girls, tend to become negative from age 11 onwards. Teachers need to have developed 'reasoning patterns' (Karplus, 2003, pp S54) so that they are able to help students develop more advanced reasoning patterns instead of enabling students to carry out work that "emphasises completion over comprehension"

(Roth & Roychoudhury, 2003, pp S115). Karplus (2003) argues that teachers should themselves be clear about the proper understanding of concepts in terms of links with other concepts as well as their mathematical relationships because the need to develop necessary core ideas are vitally important in any attempt that leads to the widening of the cognitive perspective in the formation of true scientific knowledge. This will eventually enable concept acquisition with a greater sense of certainty by the students. In other words, students need to be guided how to incorporate chunks of information acquired from different subject areas into a single body of knowledge.

Heselden and Staples (2002) are of the opinion that students learning English are unable to perform because they do not readily transfer acquired skills between subjects and that "science departments need to become involved in actively teaching pupils reading skills as well as using reading to help pupils learn", (p. 52). The classroom transactions not only depend on how teachers conceptualise their roles, but also how students perceive and conceptualise their learning (Roth & Roychoudhury, 2003). This implies that there should be shared dynamism of understanding between teachers and students otherwise both will be operating practically in different worlds; rote learning will therefore predominate. It is unfortunate that in such type of situation, the "banking concept of education" (Freire, 1972, p.46) where the teacher being the "depositor" and the students – the "depositories" will certainly prevail. Although students may merely pass examinations by adopting rote learning concepts, it doest not necessary mean that there has been integration or understanding of knowledge (Trumbull, 1999).

But to be able to understand physics without recourse to mathematical tools is an impossible task. Mathematics lies at the core of any scientific knowledge and understanding (Stevens & Lenton, 1999; Ramma & Bessoondyal, 2001;

Parmessur et al., 2002; Ramma et al., 2004) as it provides science with the necessary tools in the analysis of data. Ramma and Bessoondyal (2001) have shown that mathematics is a necessary and sufficient condition for students, taking part in the Higher School Certificate level of the Cambridge UCLES examination, to obtain good results in physics. It is not a surprise that physics is a blacklisted subject for students at 'A' level and even at post 'A' level (Osborne et al., 1998). In their analysis, Ramma & Bessoondyal (2001) argue that students have encountered many equations during their mathematics lessons but when such similar equations are encountered during their physics lessons, links are not made and students' confidence in pursuing the subject further, falters. The authors go even further, mentioning that the links can also be facilitated by the mathematics teacher. Pedersen and Yerrick (2000) clarify that science teachers has the obligation to integrate among subjects for posing and problem solving of real life problems.

The "communication gap" (Schon, 1987, p. 101) which exists between teacher and learner will widen if "reciprocal reflection-in-action" (p. 101) is not sustained by both. On one hand, Lewis (1974) is of the opinion that "cooperation is called for between the teaching, and preferably between the teachers, of the two subjects [mathematics and physics] for the benefit of the pupils" (p 155). On the other hand, Leonard et al. (1999) highlights that though students are able to solve lots of problem, they do not necessarily develop problem solving skills. Many students use a formula-driven approach to solve physics problems that does not rely on understanding underlying physics concepts and that does little to encourage the problem-solving skills employed by experts. This raises serious concern as it implies that students are involved in superficial and approximate learning rather than deep and concrete learning. There is a need to point out that prior knowledge and beliefs play a very important part in students' construction of mental models (Eilam, 2004) and with this perspective in mind it follows that students who have developed mathematical cognitive structures will be able to operate at a different mental level. Furthermore, Eilam (2004) concludes that though the same concept is being introduced, but which comprises of different levels of operation, students face difficulty in shifting between levels of operation. This is in conjecture with Lewis & Linn (2003) findings that students most of the time rely on intuitive conceptions to explain events. If such an approach is not sanctioned, students will continue to make bad choices.

It is worth noting that even at university level, the physics-mathematics problem is still prevalent. Gill (1999) clarifies that:

> "the students do not have sufficient mathematical skills for the physics/engineering courses" and that " those students who do cope with the mathematics course are still unable to apply it in context" (p.82). It is really an issue of concern when students claim that there is "no apparent relationship between the two subjects either at A-level or at university" (Gill, 1999, p. 85).

If students cannot apply the mathematics in-context, is it not a problem of teaching/learning and communication between the mathematics and physics educators at all level?

The Interrelationship between mathematics and physics – Analysis of the physics 'A' level curriculum

The 'A' curriculum (GCE Advanced Level 9702) offers a wide repertoire of mathematics' involvement in physics. For the purpose of this research, it has been found appropriate to conduct an analysis of the 'A' level curriculum and to determine the related mathematical implications at lower levels.

1. The Chapter on ***Physical Quantities and Units*** introduces the concept of physical quantities which consists of a numerical magnitude and a unit. In mathematical interpretation this statement can be

expressed as $y = k\,x$, where y is the physical quantity, k- a numerical magnitude, while x – the unit.

Homogeneity of physical equations: To verify whether an equation is homogeneous, students should have rightly acquired skills in carrying out operations involving indices in Form IV-V.

Vectors: In mathematics (Form III-V) students encounter problems in vectors; they are involved in solving problems using free and position vectors. A vector is embodied in physics in a variety of ways, such as a force, a velocity, an acceleration, etc… A vector is comprised of a length and a direction. Many problems in physics can be solved when resort to a vector approach. The equation $F = ma$ (Newton's 2nd law) involves a number of items; it is a vector equation, the acceleration will act in the same direction as force, a negative sign will have to be included if they are in opposite direction.

2. The Chapter on ***Measuring Techniques*** introduces the concept of errors. Students have developed competencies in Additional Mathematics on absolute error and fractional error. This doest not mean that students will be able to easily transfer the acquired mathematical skills into physics provided they have developed core constructs. Moreover there is a need to point out that "meaning is created in the mind of the student's sensory interaction with her or his world" (Saunders, 1992, p. 136) and since meaning is created in the mind of the learner, the only way teachers can determine whether knowledge structures have formed, is to adopt an inquiry based approach and to incorporate changes in their teaching strategies (Appleton, 1993).

3. The Chapter on ***Kinematics*** covers a wide range of mathematical skills, such as identifying linear equations (speed = distance/time, acceleration = velocity/time). From definition of acceleration, $a = \frac{v - u}{t}$

is tantamount to $v = u + at$, which is a linear equation similar to $y = mx + c$, in particular, $y = c + mx$, where *a,* equivalent to *m,* is the gradient of the *v-t* graph. Students should be able to develop understanding that this equation is linear provided acceleration is constant.

Graphs: Students have drawn many graphs in mathematics (Form III-V), but in physics understanding of relationship is very important. Lenton et al. (2000) consider "ability to handle mathematical concepts confidently is a key skill for pupils studying science" (p 15). Learners usually face difficulty in determining which type of relationship they should draw and this partly due to a lack of skills in controlling variable. Kanari and Millar (2004) affirm that students experience difficulty in interpreting graphs since they tend to "hold on to initial hypothesis in the face of disconfirming evidence" (p 760). Moreover, the authors found out that students did not anticipate problems in manipulation of variables, that is, the dependent variable and the independent variable were correctly sorted out but they had much difficulty in interpreting data, in particular in deciding whether a variable has increased or decreased or stayed constant. It is required that learners be able to link their mathematical knowledge with physics and to stand back and think to be able to correctly jump to conclusion.

Equation of Motion: (i) $v^2 = u^2 + 2as$ is the equation of a parabola if v is plotted against s, but the same equation may be linear if v^2 is plotted against s. Again the mathematical aspect of the graphs should be resorted to. (ii) $s = ut + \frac{1}{2}at^2$ is again a parabolic equation if *s* is plotted against *t* provided the accompanying variables are constants.

Projectile motion: This component requires sound knowledge of vectors in terms of components of vectors. Students should be able to consider motions of an object in two perpendicular directions independent from each other.

4. The Chapter on *Dynamics* offers a wide perspective in applying vector dimension in problem solving. For example, weight W = mg, momentum $p = mv$ or rate of change of momentum, $\frac{dp}{dt}$.

5. The Chapter on ***Forces*** involves a number of mathematical components such as forces in equilibrium can be determined using a vector triangle. A number of studies (for eg. Saljo, 1987; Tao & Gunstone, 1999) have shown that students hold misconception in interpreting the concept force. This is due to prior conception which does not have a firm theoretical (mathematical) backup. It is believed that the motion of an object is the result of a force which is stored up in the object which eventually causes the object to move or stop (Saljo, 1987).

6. Chapter - ***Work, Energy, and Power***: the definition for work done involves 'dot product' of two vectors which result in a scalar quantity, that is, ***a·b =*** | ***a*** | x | ***b*** | cos θ. Similarly, power (= *Fv)* involves the dot product of two vectors.

   The concept of Energy is also misunderstood and students have the impression that energy can be stored or can change its shape. For example wherever we use the formula for kinetic energy, $E_k = \frac{1}{2}mv^2$, it is meant that $\frac{1}{2}mv^2$ is the amount of work that an object will perform on another object or system (Beynon, 1994).

7. Chapter – *Motion in a Circle*: The derivation of angular velocity originates from Additional Mathematics (differentiation, Form IV-V) which means that students should have developed a sound understanding of the notion of angular displacement to be able to fully understand the concept. To be able to understand why centripetal

acceleration acts towards the centre of the circle, students need a vector analysis of the motion. Parmessur et al. (2002) showed that students, in particular girls, can develop critical thinking if they adopt a "*step-by-step*" (Chaiklin, 1987, p 73) instruction based on a vector approach while linking mathematics with the physics.

8. Chapter – ***Gravitational field***: The equation of Newton's law of Gravitation $F = \frac{GmM}{r^2}$ is quite problematic as it does not show the vector dimension. To be noted that force is a vector quantity, whereas all the quantities on the right side are scalar. With the mathematics perspective, the equation could be rewritten as $\vec{F} = \frac{GmM\,\vec{r}}{r^3}$, or $\vec{F} = \frac{GmM\,\hat{r}}{r^2}$. The ability to write an equation in different perspectives will certainly develop cognitive "knowledge structures" (Mintzes et al., 1998, p. 10). The manipulation of the formula enables learners to undergo *minds on* experience.

   The same chapter refers to the equation of potential at a point, $\phi = -\frac{GM}{r}$. It is interesting to note that the negative sign included in the equation has an important physical interpretation unlike the case for mathematics.

9. Chapter – ***Phases of Matter***: density, $\rho = \frac{m}{v}$, can be expressed in a linear equation similar to $y = mx$. Similarly, pressure can be expressed in similar mathematical equations and students need to be able to develop skills in manipulation and control of variables.

10. Chapter: - ***Deformation of Solids***: The notion of force is again encountered, where Hooke's law is involved. $F = ke$ is a linear equation in F and *e* where *k* is the constant of proportionality. The strain

energy, $E = \frac{1}{2}ke^2$, can either be a linear or quadratic relation depending on which variable(s) is/are plotted.

In this chapter students should be able to interpret force-extension graphs for different materials and a sound knowledge of mathematics is required.

11. Chapter – ***Ideal Gases***: The interpretation of the equation of state, $PV = nRT$, requires sound mathematical skills. Raw (1998) argues that in order to help students develop appropriate mathematical skills in physics is to give "extra tuition in the maths needed for A-level physics – for example simple algebra, graph work, trigonometry, standard form and significant figures" (p. 99). The equation of state comprises of four variables, namely *P, V, n* and *T*. Students should have developed appropriate conceptual framework and core constructs to be able to appreciate any relationship between them. At the same time the above formula can be proved using mathematical knowledge by combining the three equations $PV =$ constant, $\frac{P}{T} =$ constant, $\frac{V}{T} =$ constant.

    The equation expressing pressure exerted by a gas, $p = \frac{1}{3}\frac{Nm}{V} < c^2 >$ again incorporates a number of variables which should be controlled if the relationship is to expressed graphically.

12. Chapter – ***Temperature***: The equation $X = \frac{X_\theta - X_0}{X_{100} - X_0} x 100^0 C$ is derived based on linearity between change in length of the mercury column and temperature.

13. Chapter – ***Thermal Properties of Materials***: The equation for heat capacity and specific heat capacity are linear equations ($E = C\theta$, $E = mC\theta$) provided some variables remain constant.

The equation of the first law of thermodynamics $Q = U + pdV$ incorporates a number of constructs, namely linearity, dimension consistency and control of variable(s). The mathematical interpretation is very much required since any of the above variables may be assigned a negative sign.

14. Chapter – ***Oscillations***: Most of the concepts involved in this chapter are deeply rooted in mathematics. Students should have developed appropriate skills in trigonometry, mainly in trigonometric functions ($y = \sin(wt \pm \varphi)$, $y = \cos(wt \pm \varphi)$) and curve sketching. It is of utmost importance that the phase φ should be clearly understood; in mathematics students use the concept 'angle' instead. The phase of a particle undergoing simple harmonic motion is expressed as $\varphi = \frac{2\pi x}{\lambda}$, and students needs to develop adequate skills in interpretation of it. They need to pay particular attention that *x* and λ should be in phase.

The defining equation for simple harmonic motion, $a = -\omega^2 x$ is a linear one when expressed graphically, provided one of the variable remains constant. Students should have developed adequate skills when proving that $a = -\omega^2 x$ is a solution to the equation $x = x_0 \sin \omega t$ or $x = x_0 \cos \omega t$. They also need to identify when the relationship is a cosine or sine one. If learning in mathematics is meaningful, then students will encounter no difficulty in physics.

Students should also be able to show that the velocity-displacement relationship is ellipsoid ($v = \pm\omega\sqrt{(x_0^2 - x^2}$).

Graphical relationships of changes in displacement, velocity and acceleration during simple harmonic motion are example of the wide context of mathematics requirement in this chapter.

Students are expected to show the interchange between kinetic energy ($E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$) and potential energy ($E_p = \frac{1}{2}m\omega^2 x^2$) in graphs form. Students should recognise that the two equations are equations of parabola, but with different shapes since they have encountered similar equations in mathematics (or additional mathematics). When the two equations are added students should infer that the total energy is constant. Are the students able to recognise that? The answer is yes provided they have developed core constructs.

Moreover, students should be able to describe graphically how the amplitude of a forced oscillation changes with frequency. So, students' ability to draw and interpret graphs is a key factor to meaningful learning in physics.

15. Chapter – ***Waves***: Students should be able to use the equation for speed of a wave, $v = f\lambda$ in specific context and be able to manipulate the variables.

    The relationship of intensity α (amplitude)^2 requires students to make association with a parabolic relationship.

16. Chapter –***Superposition***: The principle of superposition is a directly rooted in vector addition, a topic encountered in lower secondary mathematics.

    Students are expected to derive the equation $\Delta x = \frac{\lambda D}{a}$ when applying geometry. Moreover, $d\sin\theta = n\lambda$ involves a number of variables which should be controlled if relationships are to be determined.

17. Chapter – ***Electric Fields***: The concept of electric field is related to a vector line (field lines) with given magnitude and direction. The relationship between force and field strength is linear (F = EQ). The electric field strength E = $\frac{dV}{dr}$ may be constant or not, depending on the

field vector. A sound knowledge of mathematics and physics is thus required for the interpretation part.

In calculating the motion of charged particles in uniform fields, students require a sound knowledge of mathematics and physics which incorporates projectile motion.

Coulomb's law $F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ is similar to Newton's law of gravitation and can be expressed in vector form.

18. Chapter – ***Capacitance***: The equation $C = \frac{Q}{V}$ involves more that one variable and students should be able to control the variables for drawing relationship.

    $I = \frac{dQ}{dt}$ refers to the rate of change of charge; its principle has already been covered in Additional Mathematics.

    Students are expected to explain characteristics for ohmic and non-ohmic conductors (Form IV-V). A sound knowledge of differentiation is required.

19. Chapter – ***D.C. Circuits***: Students are expected to solve problems involving Kirchhoffs' laws. A sound knowledge of vector addition is required. In this chapter students should be able to solve simultaneous equations involving abstracts quantities such as current, potential difference or resistance. The direction (sign) of current flow should be taken into consideration.

20. Chapter – ***Magnetic Fields***: Field lines are considered; magnitude and direction.

21. Chapter – ***Electromagnetism***: The equation $F = BIl\sin\theta$ or $F = BQv\sin\theta$ involve a number of variable which should be controlled if relationships are to be obtained.

22. Chapter – ***Laws of Electromagnetic Induction***: The equation $\Phi = AB$ incorporates three variables. For graphical representation, one variable should be controlled.

Faraday's law $F = -\frac{d\Phi}{dt}$ refers to rate of change of flux linkage (mathematical relationship) and the negative sign has physical significance.

23. Chapter – ***Alternating Currents:*** The derivation of the root-mean-square requires sound mathematical skills as it involves trigonometric functions.

Students are expected to draw half-wave and full-wave rectification. Adequate graphical skills are needed.

24. Chapter – ***Charged Particles***: In investigating the quantisation of charge, knowledge of mathematics is required (net force $\neq 0$). Furthermore, the determination of velocity selector requires mathematical skills.

25. Chapter – ***Quantum Physics***: The equation $E = hf$ is a linear equation in *E* and *f*. The equation $hf = \Phi + \frac{1}{2}mv_{\max}^2$ comprises of numerous variables and students are expected to develop skills in controlling them to obtain required relationship.

The *de Broglie* equation $\lambda = \frac{h}{p}$ is an inverse relationship in λ and *p*.

The ability to solve problems using the relation $hf = E_1 - E_2$ requires mathematical competence.

26. Chapter – ***Nuclear Physics***: Students are expected to apply simple mathematical addition/subtraction to represent simple nuclear equations.

The equation for activity of a radioactive element, $A = \lambda N$ is linear. Students should use knowledge of mathematics to solve problems using the exponential nature of radioactive decay, $x = x_0 e^{-\lambda t}$ and should be able to draw exponential relationship.

Analysis of the Mathematics Curriculum

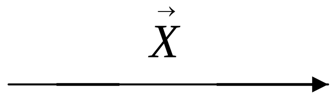
This section provides an insight into the mathematics concepts that students encounter at the secondary level of education. The analysis of the mathematics and additional mathematics curricula has been made with direct reference to the physics curriculum.

Mathematics F3 & Mathematics SC

Students are expected to develop skills in interpretation of graphs in mathematics. However, plotting of graph is essential in the chapter motion in physics. Students should have developed a sound knowledge of gradients in mathematics. Graphs of acceleration, speed, velocity distance and displacement against time are frequently included in examination. These graphs may be straight lines or curves.

Mathematics F2, Vectors, F3

This is a very important concept in mathematics as well as in physics. Many problems in physics can be solved if a vector dimension is adopted. In mathematics, vector quantities are represented by a section of a straight line whose length represents the magnitude of the vector and whose direction represents the direction of the vector.



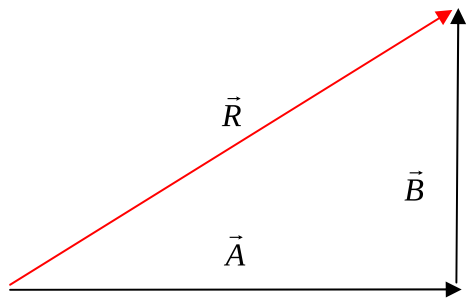
Mathematics F2, Addition of vectors, F3

Two vectors can be added by using the vector addition. $\vec{R}$ is the vector sum of the vectors $\vec{A}$ and $\vec{B}$. It is termed the resultant of the two vectors.

$$\vec{R} = \vec{A} + \vec{B}$$

To represent the addition of vectors geometrically, we must:

1. Draw vectors $\vec{A}$ followed by vector $\vec{B}$ respecting the orientation of both vectors
2. Draw the resultant vector from the 'starting position' to the 'final position'



It is important that students understand that though vector addition is commutative, they should as a matter of principle, always add two vectors as per the correct order of the mathematical statement.

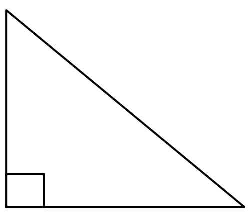
Mathematics F2, Square roots, cube roots and applications

The three vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$ form a right angle triangle and hence Pythagoras theorem is applied to calculate the magnitude of the resultant vector $\vec{R}$ (denoted by $|\vec{R}|$).

The Pythagoras theorem states that the square of the longest side of the right-angled triangle is equal to the sum of the square of the other two sides.

For the triangle drawn below, Pythagoras theorem gives

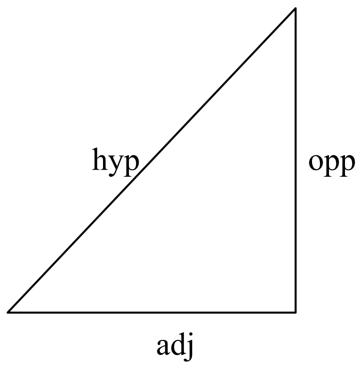
$$z^2 = x^2 + y^2$$



Thus, using Pythagoras theorem, $|\vec{R}| = \sqrt{A^2 + B^2}$ where A and B represent the magnitude of the vectors $\vec{A}$ and $\vec{B}$.

Mathematics F3, cosine, sine, tangent

Consider the right angled triangle below with at angle θ given.



The sides can be labelled as *hypotenuse (hyp)*, *adjacent (adj)* and *opposite (opp)* according to the position of the angle θ.

Trigonometric functions (such as sin, cos & tan) can be obtained in terms of the sides of the right angle triangle

1. $\cos\theta = \dfrac{adj}{hyp}$

2. $\sin\theta = \dfrac{opp}{hyp}$

3. $\tan\theta = \dfrac{opp}{adj}$

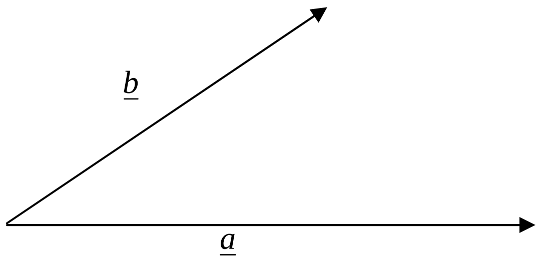
Equation (3) is exploited to find the angle that the resultant makes with the vector $\vec{A}$. This specifies the direction of the resultant vector $\vec{R}$.

Additional Mathematics, School Certificate

In calculus, $\frac{\delta y}{\delta x}$ is approximately equal to $\frac{dy}{dx}$ only when the change is small.

Additional Mathematics, School Certificate

Any vector can be projected onto another one.



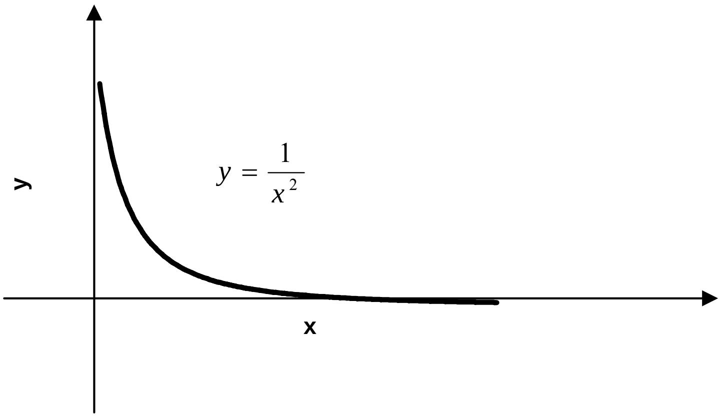
Projection of $\underline{b}$ on $\underline{a}$ is given by $|b|\cos\theta$ where θ is the angle between the two vectors. Using this method, vector v is projected onto the x and y axes. Thus, v is resolved into two components $v\cos\theta$ and $v\sin\theta$ along the x and y axes respectively.

Mathematics, School Certificate

The equation $F \propto \frac{Mm}{r^2}$ is similar to $y \propto \frac{1}{x^2}$. This means that y varies inversely as the square of x. Thus an increase in x by a factor of 5 leads to a decrease by factor of 5^2. Alternatively, the equation may be written as $y = \frac{k}{x^2}$ where k is the constant of proportionality. Thus the equation $F = \frac{GMm}{r^2}$ may be expressed as $F = \frac{k}{r^2}$ where GMm is the constant of proportionality.

Mathematics, School Certificate

The graph of F v/s r obeys the inverse square law and should resemble that of $y=\frac{1}{x^2}$. $x=0$ and $y=0$ Are both asymptotes in the latter graph which is sketched for $x \geq 0$ below.



Mathematics F3, Further equation and change of subject of formula

Change of subject of equation must be performed before computing the unit of k.

$$F=\frac{kQ_1Q_2}{r^2}$$

$$k=\frac{Fr^2}{Q_1Q_2}$$

Mathematics F3, Indices

Simplification of the equation $[k]=\frac{Nm^2}{c^2}$ requires the use of indices. The laws of indices and other properties are listed below:

1. $a^p \times a^q = a^{p+q}$
2. $a^p \div a^q = a^{p-q}$

3. $\left(a^{p}\right)^{q} = a^{pq}$

4. . $a^{n} \times b^{n} = (ab)^{n}$

5. $\dfrac{a^{n}}{b^{n}} = \left(\dfrac{a}{b}\right)^{n}$

6. $a^{0} = 1$

7. $a^{-p} = \dfrac{1}{a^{p}}$

8. $a^{\frac{1}{p}} = \sqrt[p]{a}$

9. $a^{\frac{p}{q}} = \left(a^{\frac{1}{q}}\right)^{p}$ or $\left(a^{p}\right)^{\frac{1}{q}}$

$= \left(\sqrt[q]{a}\right)^{p}$ or $\left(\sqrt[p]{a}\right)^{q}$

In this case, we make use of the fact that $a^{-p} = \dfrac{1}{a^{p}}$.

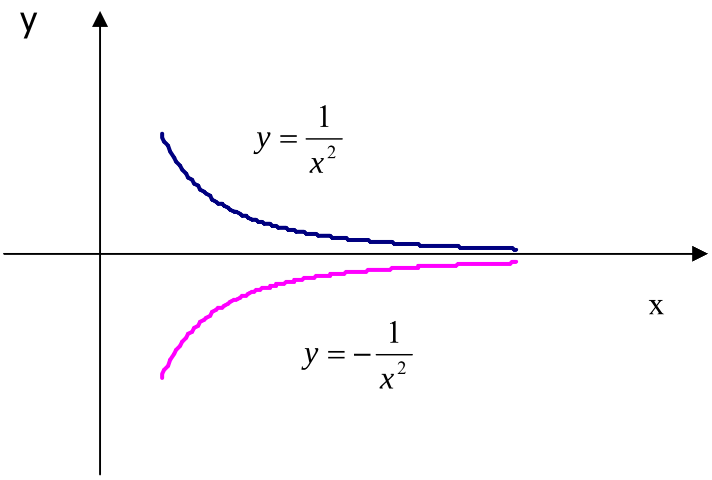
Change of subject of equation (G subject)

Indices $\dfrac{N}{C} = NC^{-1}$ $\quad a^{-p} = \dfrac{1}{a^{p}}$

Mathematics, School Certificate

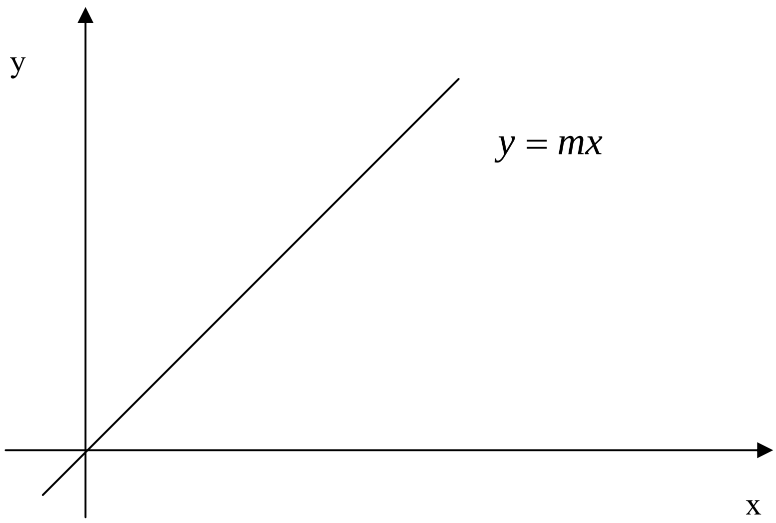
The graph of F v/s r for attraction between two opposite charges corresponds to the graph of $y = \dfrac{1}{x^{2}}$

Repulsion between like charges is analogous to the graph of $y = -\dfrac{1}{x^{2}}$ which is just the reflection of $y = \dfrac{1}{x^{2}}$ about the x axis. Figure 1 shows both graphs for $x \geq 0$.



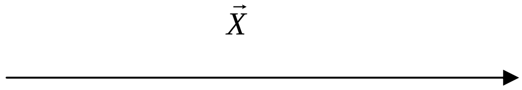
Mathematics F2, coordinates, F3, Coordinates

The equation of a straight line passing through the origin and having constant gradient m is in the form $y = mx$. Thus, both $F = mg$ and $s = ut$ are equations of straight lines with gradient m and u respectively and y intercept 0.



Mathematics F2, Vectors, F3

Vector quantities can be represented by a section of a straight line whose length represents the magnitude of the vector and whose direction represents the direction of the vector.

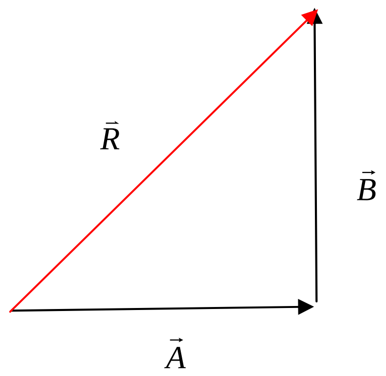


Mathematics F2, Addition of vectors, F3

The addition of two vectors requires adaptation of appropriate skills. $\vec{R}$ is the vector sum of the vectors $\vec{A}$ and $\vec{B}$. It is termed the resultant of the two vectors. $\vec{R} = \vec{A} + \vec{B}$

To represent the addition of vectors geometrically, we must:

3. Draw vectors $\vec{A}$ 'followed by' vector $\vec{B}$ respecting the orientation of both vectors
4. Draw the resultant vector from the 'starting position' to the 'final position'



Mathematics F2, Square roots, cube roots and applications

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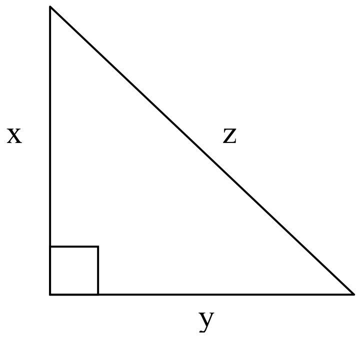
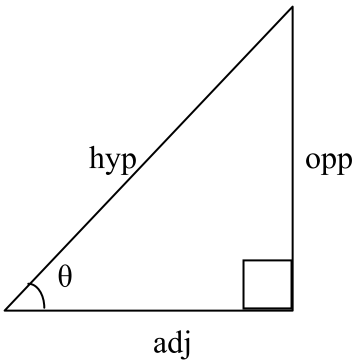


Thus, using Pythagoras theorem, $\left|\vec{R}\right| = \sqrt{A^2 + B^2}$ where A and B represent the magnitude of the vectors $\vec{A}$ and $\vec{B}$.

Mathematics F3, cosine, sine, tangent

Consider the right angled triangle below with at angle θ given.



The sides can be labelled as *hypotenuse (hyp)*, *adjacent (adj)* and *opposite (opp)* according to the position of the angle θ.

Trigonometric functions (such as sin, cos & tan) can be obtained in terms of the sides of the right angle triangle

1. $\cos\theta = \dfrac{adj}{hyp}$

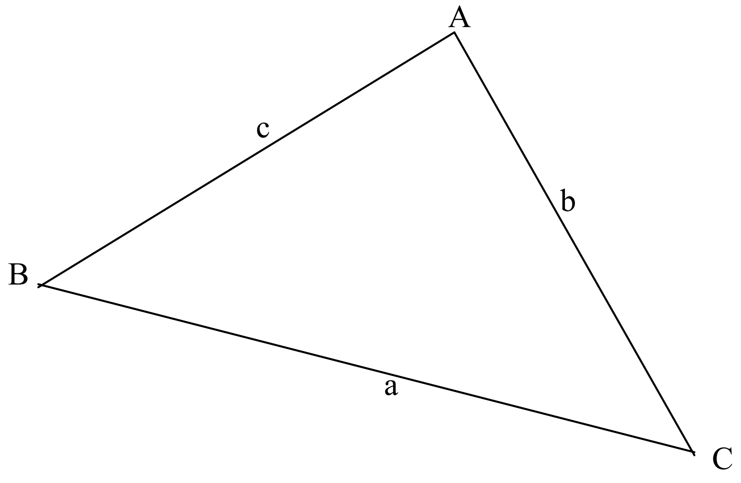
2. $\sin\theta = \dfrac{opp}{hyp}$

3. $\tan\theta = \dfrac{opp}{adj}$

Equation (3) is exploited to find the angle that the resultant makes with the vector $\vec{A}$. This specifies the direction of the resultant vector $\vec{R}$.

Mathematics, School Certificate

The cosine and sine rule are particularly useful to calculate an unknown angle or an unknown side in triangles with no right angles.



The cosine and sine rules read as follows:

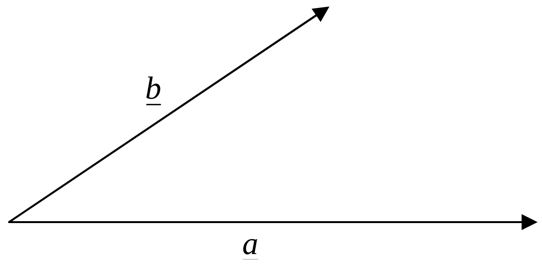
1. $a^2 = b^2 + c^2 - 2bc\cos A$

2. $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

When two vectors $\vec{A}$ and $\vec{B}$ (which are not mutually perpendicular) are added, the two equations above can be used to determine the magnitude and direction of the resultant vector.

Additional Mathematics, School Certificate

Any vector can be projected onto another one.



Projection of $\underline{b}$ on $\underline{a}$ is given by $|b|\cos\theta$ where θ is the angle between the two vectors. Using this method, vector F is projected onto the x and y axes. Thus, F is resolved into two components $F\cos\theta$ and $F\sin\theta$ along the x and y axes respectively.

Mathematics F3, Cosine sine tangent

Using trigonometry

The table below gives the sines, cosines and tangents of common angles used in physics and mathematics.

	0^o	30^o	45^o	60^o	90^o	180^o
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0

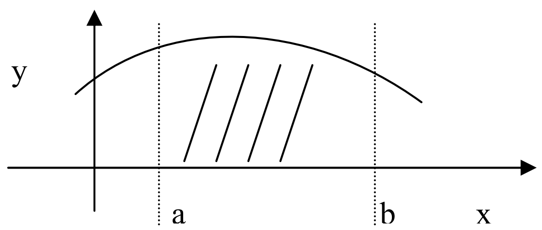
The work done, given by the equation $W = Fs\cos\theta$, can be adapted to special cases where θ =0°, 90°, or 180° by using the above table.

Additional Mathematics, School Certificate

A small change in variable x is denoted by δx or Δx

Additional Mathematics, School Certificate

The shaded area can be estimated splitting the area into thin vertical strips and treating each strip as being approximately rectangular. Once the area is divided into thin strips (the thinner the strips are, the better the approximation), the area of each strip is determined and summed to obtain the total area.



The area δA for one strip of width δx and height y is given by $\delta A = y\delta x$.

Thus, the total area is $A = \lim_{\delta x \to 0} \sum_{x=a}^{x=b} \delta A = \lim_{\delta x \to 0} \sum_{x=a}^{x=b} y\delta x$

In a similar fashion, the work done by the variable force *F* over the displacement *s* is calculated by splitting the area under the graph into narrow strips. The area of the strips are determined (area of shaded strip= $F\delta x$) and summed to give the total area.

Additional Mathematics, School Certificate

Differentiation can be summarised by the following formulae:

1. $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

2. $\frac{d}{dx}\left(ax^n\right) = a\frac{d}{dx}\left(x^n\right) = anx^{n-1}$

3. $\frac{d}{dx}(a) = 0$

4. $\frac{d}{dx}\left(x^m + x^n\right) = \frac{d}{dx}\left(x^m\right) + \frac{d}{dx}\left(x^n\right) = mx^{m-1} + nx^{n-1}$

5. $\frac{d}{dx}(x-a)^n = n(x-a)^{n-1}$

The 2nd rule above was used to differentiate work to obtain power.

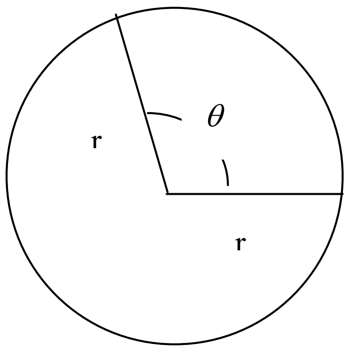
$$P = \frac{dW}{dt} = \frac{d}{dt}(Fx) = F\frac{dx}{dt} = Fv$$

Indices $\frac{J}{s} = Js^{-1}$ $\qquad a^{-p} = \frac{1}{a^p}$

Mathematics, F2

Additional Mathematics, School Certificate

The angle subtended at the centre of a circle by an arc equal in length to the radius is 1 radian (1 rad).



In general, if the length of an arc is s and the radius is r then

$$s = r\theta$$

Or

$$\theta = \frac{s}{r}$$

θ is the ratio of the arc length to the length of the radius. Thus, the angle in one complete revolution is $\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$ radians since the circumference of the circle is $s = 2\pi r$.

But, in degrees, $\theta = 360^o$. Thus, we obtain a relation between radians and degrees measures:

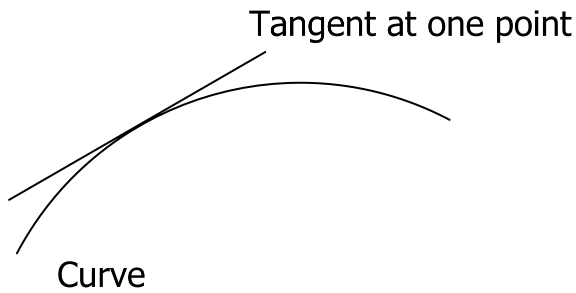
$$2\pi \ radians = 360^o$$

Mathematics, School Certificate

Drawing tangents

A tangent is a straight line which touches a curve at one point. The gradient of the tangent is equal to the gradient of the curve at that point.

A tangent drawn at a point on a circular path represents the velocity of a particle at that point.



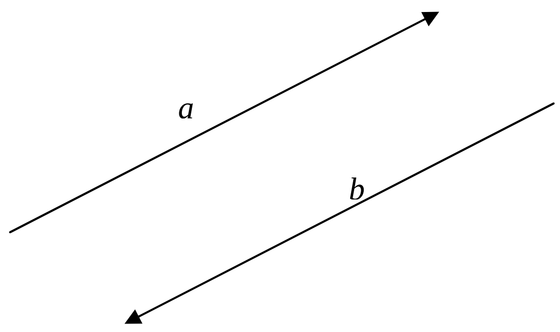
Trigonometry $\sin\theta = \frac{opp}{hyp}$ $\sin\beta$ can be approximated to β only when β is small and measured in radians. This approximation is important in the derivation of the equation $a = r\omega^2$.

Additional Mathematics, School Certificate

In calculus, $\frac{\delta y}{\delta x}$ is approximately equal to $\frac{dy}{dx}$ only when $\delta x \to 0$. Thus, in the derivation for a, $\frac{\delta\theta}{\delta t}$ can be substituted by $\frac{d\theta}{dt}$ and eventually by ω.

Mathematics, F2

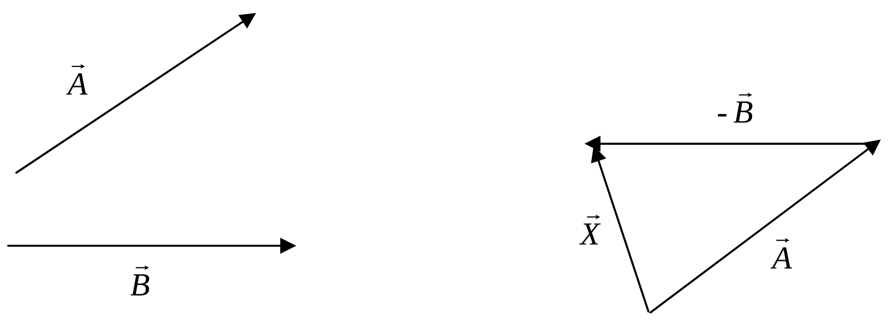
When two vectors are related as $\underline{a} = -\underline{b}$, it implies that their magnitude is the same but the direction is opposite. The velocity vectors are equal in magnitude but their direction is opposite. Thus, one writes $\underline{v}_A = \underline{v}_B$.



Mathematics, F2

Vector subtraction

$\vec{X} = \vec{A} - \vec{B}$ can be expressed as $\vec{X} = \vec{A} + \left(-\vec{B}\right)$. The latter equation means that $\vec{X}$ is the vector sum of $\vec{A}$ and $-\vec{B}$ as depicted below.



The change in velocity is the vector sum of the of the two velocity vectors $-\underline{v}_A$ and $\underline{v}_B$.

$$\left(\frac{\Delta\theta}{\Delta t}\right)_{\Delta t \to 0} = \frac{d\theta}{dt}$$ (use of calculus)

Resolution of vectors

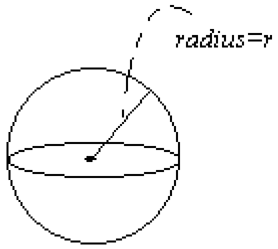
Trigonometry (sin, cos, tan)   Use of indices $\frac{N}{kg} = Nkg^{-1}$ $a^{-p} = \frac{1}{a^p}$

Graphical representation of $y = \frac{1}{x^2}$

Vector addition

Mathematics, School Certificate

The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$



Graph of $y = mx$

$y = \frac{1}{x^2}$

Use of indices $\frac{J}{kg} = Jkg^{-1}$ $a^{-p} = \frac{1}{a^p}$

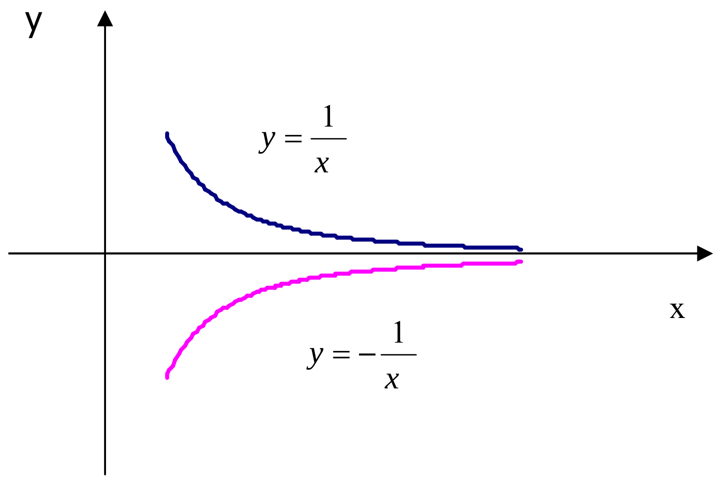
Indices $a^{-p} = \frac{1}{a^p}$

differentiation $\frac{d}{dx}\left(ax^n\right) = a\frac{d}{dx}\left(x^n\right) = anx^{n-1}$

Additional Mathematics, SC

When a function $y = f(x)$ is differentiated, $\frac{dy}{dx}$ (when evaluated at a point) represents the gradient of the graph $y = f(x)$ at that point. Hence, $\frac{dU}{dr}$ represents the gradient of the *U-r* graph.

The graph of $y = \frac{1}{x}$ has $x = 0$ and $y = 0$ as asymptotes but is less close to the axes as $y = \frac{1}{x^2}$. When this graph is reflected onto the x axis, the curve $y = -\frac{1}{x}$ is obtained which is similar to the curve of (i) *U* vs *r* *(ii)* Gpe against *r*

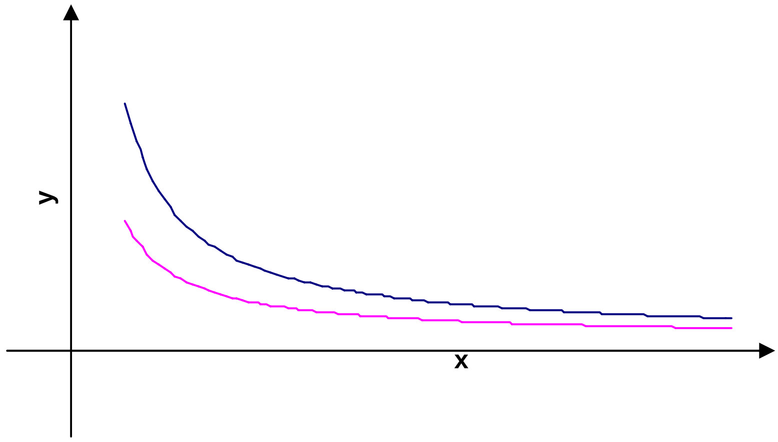


Indices $a^{-p} = \frac{1}{a^p}$

Manipulate algebraic expressions

Mathematics, School Certificate

The graph of $y = \frac{1}{2x}$ is a curve which resembles $y = \frac{1}{x}$ except that the y ordinate of the former is half that of the latter.



Mathematics, School Certificate

When a variable *x* is proportional to another variable *y*, we denote it by $y \propto x$ or $y = kx$ where *k* is the constant of proportionality. Such a relation is illustrated by a straight line passing through the origin. The gradient of the line is equal to *k* (the constant of proportionality).

Hooke's law states that *F* is proportional to *e*. Thus, in the graph of *F* vs *e*, the straight line represents the region where Hooke's law is obeyed.

Additional Mathematics, School Certificate

Integration

The process of finding a function from its derivative is called integration and it reverses the operation of differentiation. Integration is summarised below.

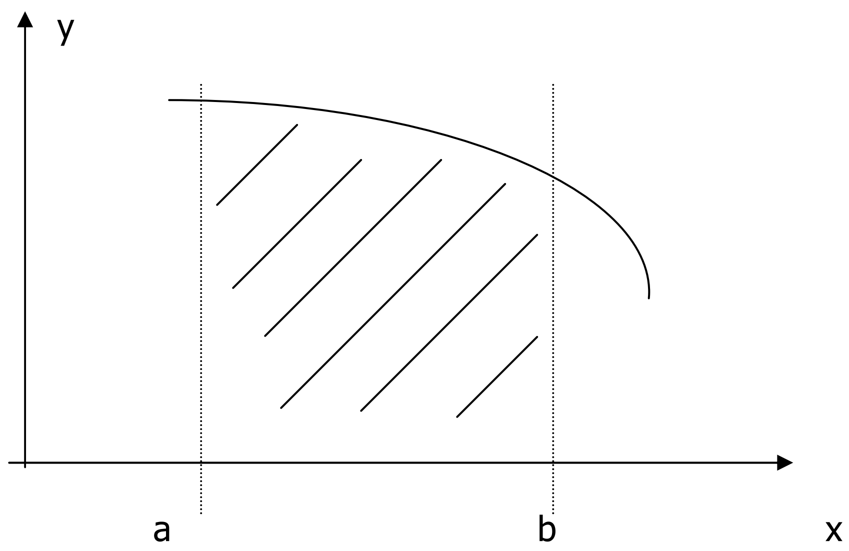
1. $\int x^n dx = \frac{1}{n+1} x^{n+1} + k$ *k*: constant of integration
2. $\int c\ dx = cx + k$
3. $\int cx^n\ dx = c\int x^n\ dx = c\frac{1}{n+1} x^{n+1} + k$

4. $\int (x^m + x^n)dx = \int x^m dx + \int x^n dx = \frac{1}{m+1}x^{m+1} + \frac{1}{n+1}x^{n+1} + k$

5. $\int (ax+b)^n dx = \frac{1}{(n+1)a}(ax+b)^{n+1} + k$

6. If $f'(x)dx = \frac{d}{dx}f(x)$ then $\int_a^b f'(x)dx = f(x)\Big|_b^a = f(a) - f(b)$

The graph below shows the variation of *y* against *x*.



As explained, the area between the graph and the *x* axis is

$$A = \lim_{\delta x \to 0} \sum_{x=a}^{x=b} y\delta x$$

But, from calculus, we have the following results

1. $\lim_{\delta x \to 0} \frac{\delta A}{\delta x} = \frac{dA}{dx} = y$

2. $\lim_{\delta x \to 0} \sum_{x=a}^{x=b} y\delta x = \int_a^b y dx$

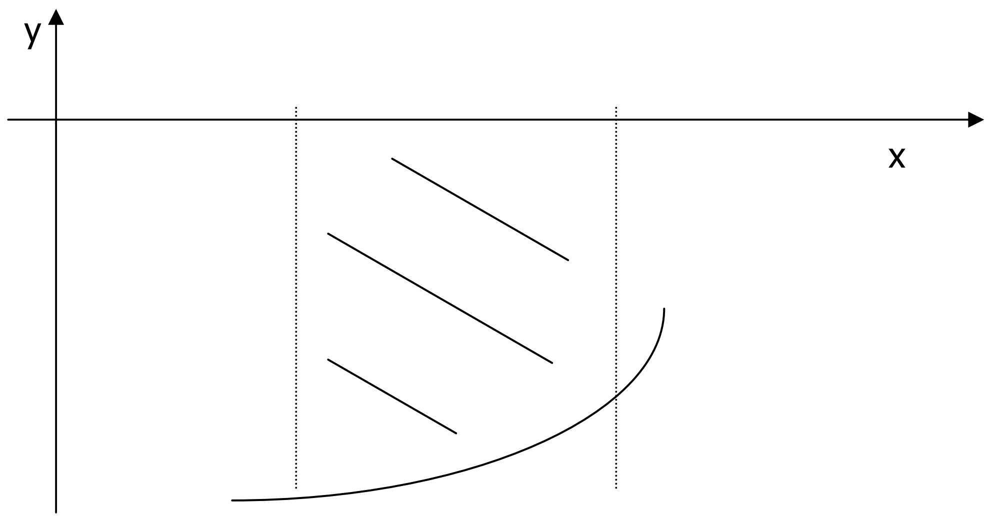
Therefore, the area between the graph and the x axis (shaded area A) is determined thru' integration.

$$A = \int_a^b f(x)dx$$

To find the total work done, we integrate *kx* w.r.t. *x* for $0 \le x \le e$

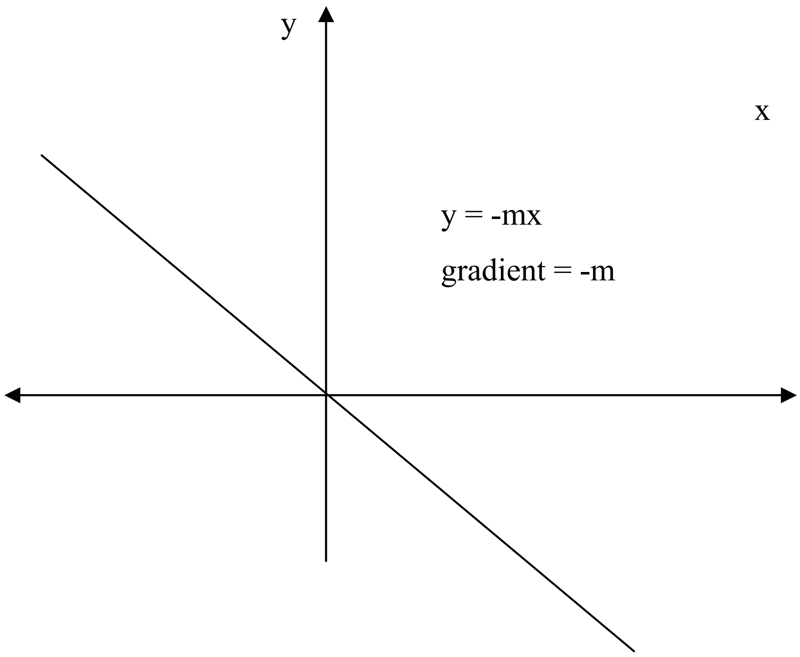
$$W = \int_0^e kxdx = \left[\frac{kx^2}{2}\right]_0^e = ke^2$$

When the area between the graph and the x axis is determined for a graph which is below the x axis for the region of interest, the area is negative.



Mathematics, F2

$y = -mx$ is a straight line passing thru' the origin as shown below. It has a gradient -*m.*

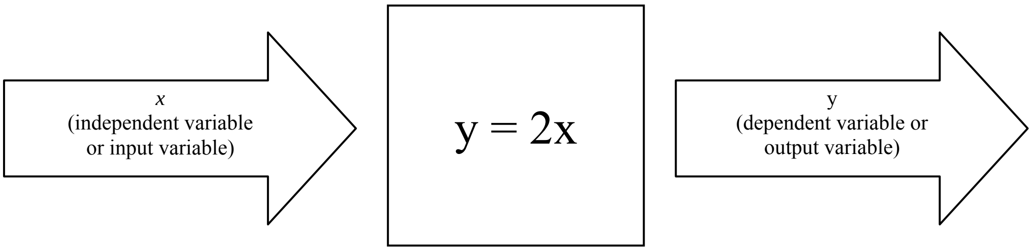


Mathematics, School Certificate

$y \propto \frac{1}{x}$ or $y = \frac{k}{x}$ means that y is inversely proportional to x. This means that if x is doubled, then y is halved. $y = \frac{k}{x}$ can be written as $yx = k$, that is, the product of the two variables is equal to a constant.

Thus, Boyle's law can be expressed mathematically as $p \propto \frac{1}{V}$ or $PV = const$ (with m and T being constant).

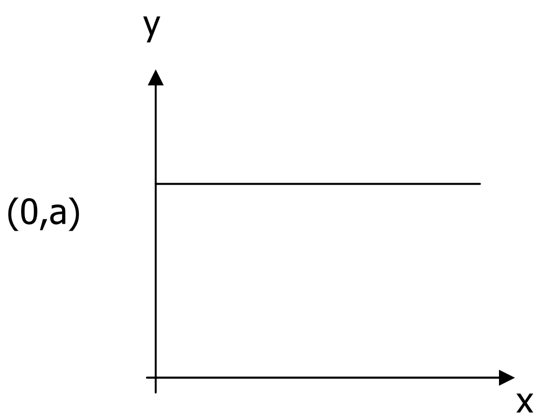
In equation $y = 2x$, x is the independent variable (input variable) and y is said to be the dependent variable (output variable).



Similarly, P and V are dependent and independent variables respectively. $(P = \frac{k}{V})$

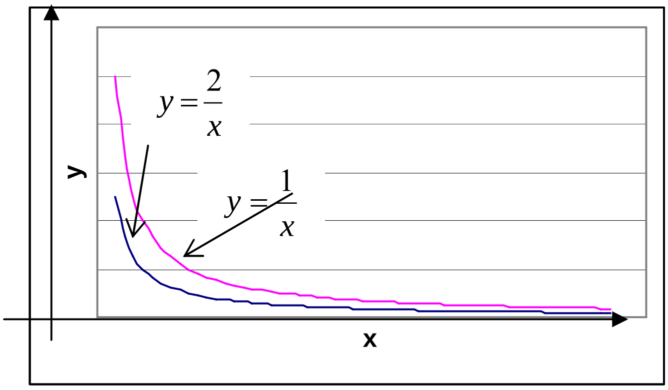
Mathematics, F2

$y = a$ is a straight line parallel to the x axis (gradient=0). The y ordinate is the same whatever the value of x.



Since $PV = k$, the graph of PV v/s P is analogous to the graph above. As the constant k is increased, the graph intercepts the y axis at a higher y ordinate.

The graph of $y = \frac{2}{x}$ has the same shape as $y = \frac{1}{x}$. However, the y ordinate for the former is twice that of the latter graph for the same value of x.



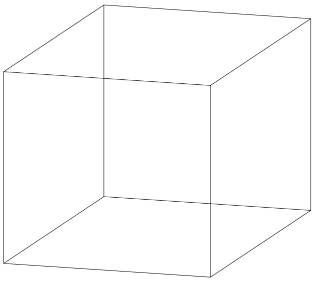
$y = mx$ *(P,T)*

$y = mx$ *(V,T)* Indices $a^{-p} = \frac{1}{a^p}$

Indices $a^{-p} = \frac{1}{a^p}$

Primary

The volume of a cube with side *r* is r^3 and the area of one side is r^2.



Use of calculus $\frac{d\vec{P}}{dt} = \frac{\Delta\vec{P}}{\Delta t}$

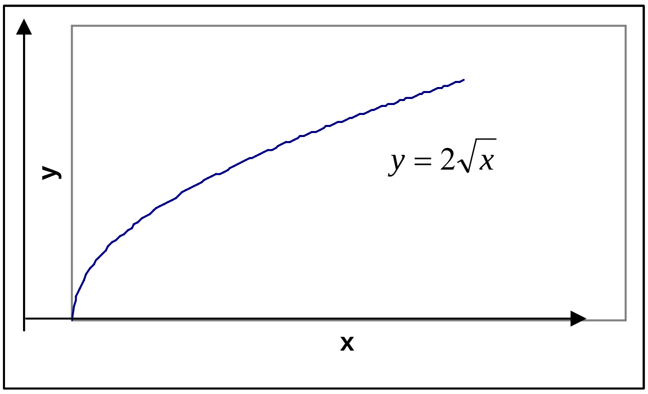
Mathematics, F3

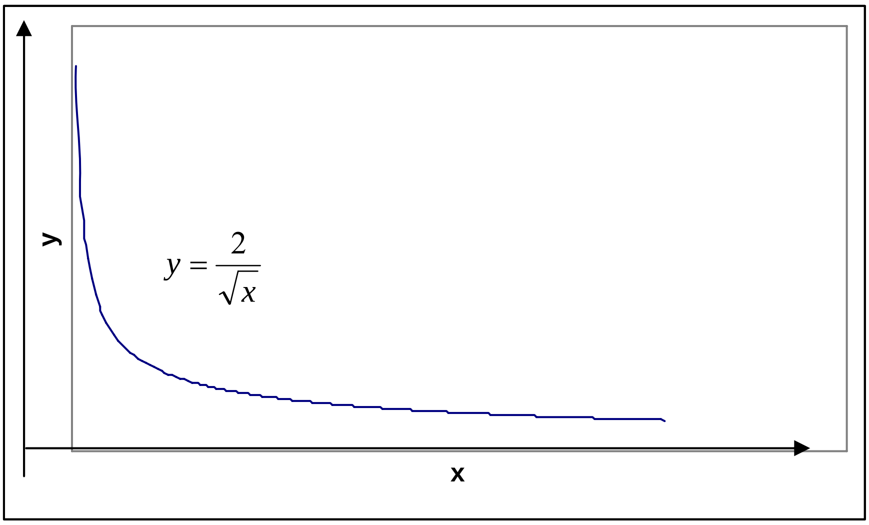
The mean value of x_1, x_2, x_3, .x_4 and x_5 is usually denoted by $\langle x \rangle$ or $\bar{x}$.

$$\langle x \rangle \text{ or } \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

Mathematics, School Certificate

The graph of $y = 2\sqrt{x}$ and $y = \frac{2}{\sqrt{x}}$ are sketched below. In these graph, the x ordinate is always positive.





The Maxwell distribution describes the distribution of particle speeds in an ideal gas. The distribution may be characterized in a variety of ways.

- **Average Speed**

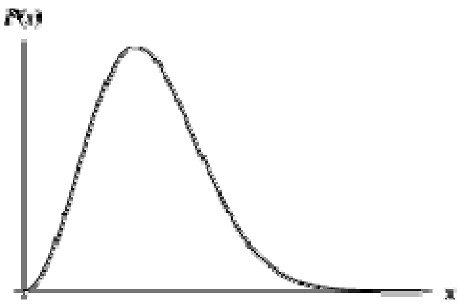
The average speed is the sum of the speeds of all of the particles divided by the number of particles.

- **Most Probable Speed**

The most probable speed is the speed associated with the highest point in the Maxwell distribution. Only a small fraction of particles might have this speed, but it is more likely than any other speed.

- **Width of the Distribution**

The width of the distribution characterizes the most likely range of speeds for the particles. One measure of the width is the **Full Width at Half Maximum (FWHM)**. To determine this value, find the height of the distribution at the most probable speed (this is the maximum height of the distribution). Divide the maximum height by two to obtain the half height, and locate the two speeds in the distribution that have this half-height value. On speed will be greater than the most probably speed and the other speed will be smaller. The full width is the difference between the two speeds at the half-maximum value.



Additional Mathematics, School Certificate

If $y = f(x)$, then

$\frac{dy}{dx} = f'(x)$ is the derivative of $f(x)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ or $f''(x) = \frac{d}{dx}\left(f'(x)\right)$ is the second derivative of $f(x)$ and is obtained by differentiating $\frac{dy}{dx}$ twice.

v is the second derivative of the displacement *x* w.r.t. *t*, whilst *a* is the derivative of the displacement *x* w.r.t. *t*.

$$v = \frac{dx}{dt} \qquad a = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

Indices *2 $a^{-p} = \frac{1}{a^p}$

Additional Mathematics, School Certificate

Differentiation of trigonometric functions

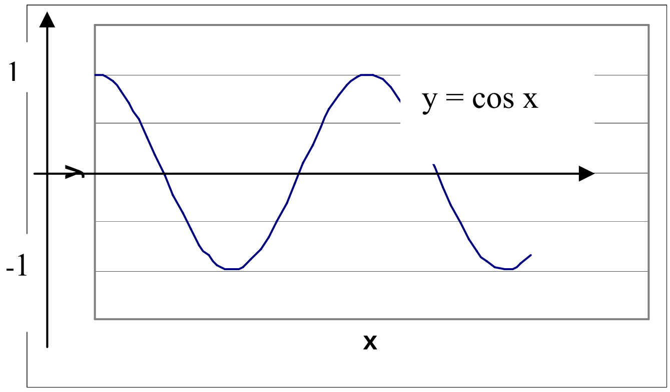
The rules of trigonometry can be used to for the differentiation of sines and cosines.

	y	$\frac{dy}{dx}$
1	$\sin x$	$\cos x$
2	$\sin(ax+b)$	$a\cos(ax+b)$
3	$\cos x$	$-\sin x$
4	$\cos(ax+b)$	$-a\sin(ax+b)$

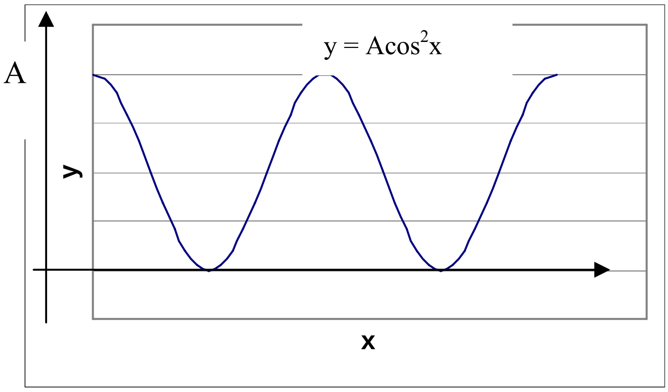
Thus, v and *a* are obtained by using (2) and (4).

Additional Mathematics, School Certificate

The graph of $y=\cos x$ consists of a basic pattern which repeats at regular interval of 2π. The graph is said to be periodic. The graph oscillates between $y=-1$ (maximum value) and $y=1$ (minimum value).



The graph of $y=A\cos^2 x$ is periodic and it repeats at regular interval of 2π. However, the graph oscillates between $y=0$ and $y=A$. It is sketched below.



Additional Mathematics, School Certificate

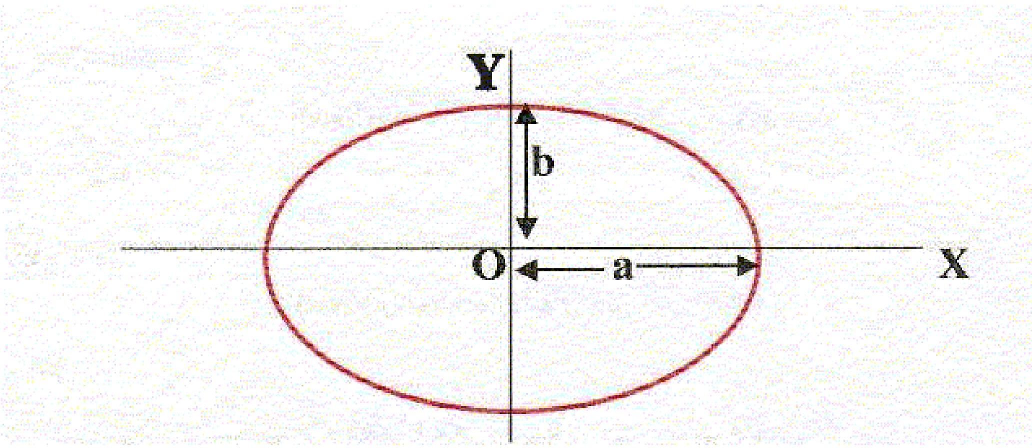
Fundamental trigonometric identities are:

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \cos ec^2\theta$$

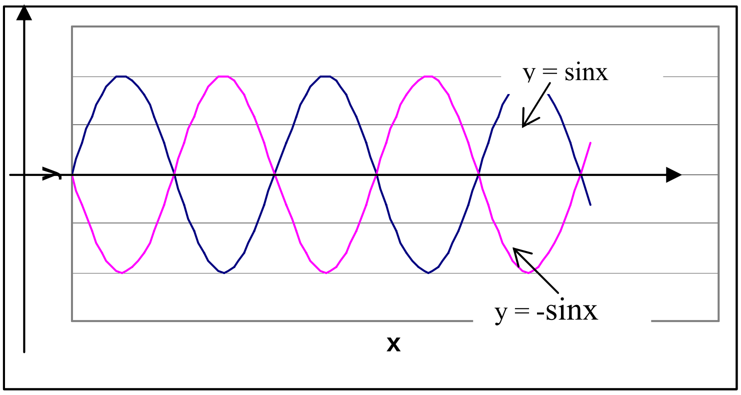
The equation representing an ellipse with centre (0,0) is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The sketch below illustrates that ellipse with $a > b$



Additional Mathematics, School Certificate

The graph of $y = \sin x$ consists of a basic pattern which repeats at regular interval and is said to be periodic. The width of the basic pattern is 2π. We note that the graph oscillates between $y = -1$ (maximum value) and $y = 1$

(minimum value). It is sketched below together with the graph of $y = -\sin x$ is just the reflection of $y = -\sin x$ about the *x* axis.



When $f'(x)$ is integrated w.r.t. *x*, the answer obtained is $y = f(x) + k$. *k* is the constant of integration. It is found by replacing a replacing a known value for *x* and the corresponding value for *y* in the equation $y = f(x) + k$.

Additional Mathematics, School Certificate

Integration of trigonometric functions

It can be shown by considering the corresponding differentiation that

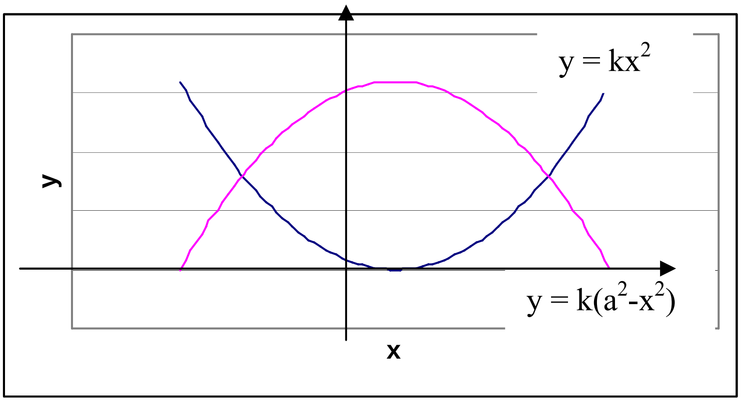
1. $$\int \cos(ax + b)dx = \frac{1}{a}\sin(ax + b) + k$$

2. $$\int \sin(ax + b)dx = -\frac{1}{a}\cos(ax + b) + k$$

Mathematics, School Certificate

$y = k(a^2 - x^2)$ and $y = kx^2$ are both the equations of a parabola. The former is obtained from the latter by using the following two transformations:

- Reflection of $y = kx^2$ about the x axis.
- Addition of ka^2 to shift the graph along the y axis by $+ka^2$ units

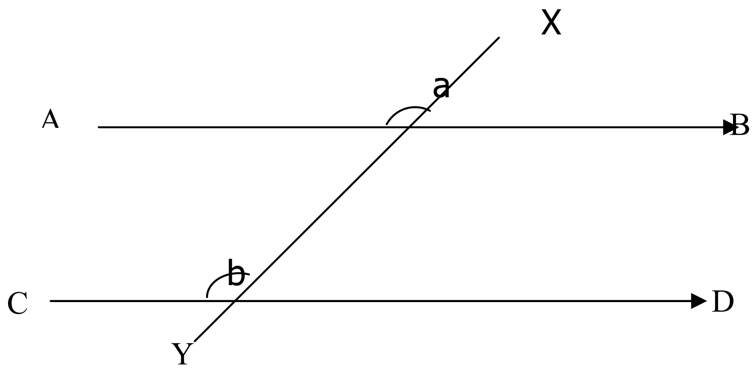


Mathematics, School Certificate

Corresponding angle

In the figure below, if AB is parallel to CD, then the angle *a* is equal to the angle *b*.

Alternatively, given that the angle *a* is equal to the angle *b,* then AB is parallel to CD.

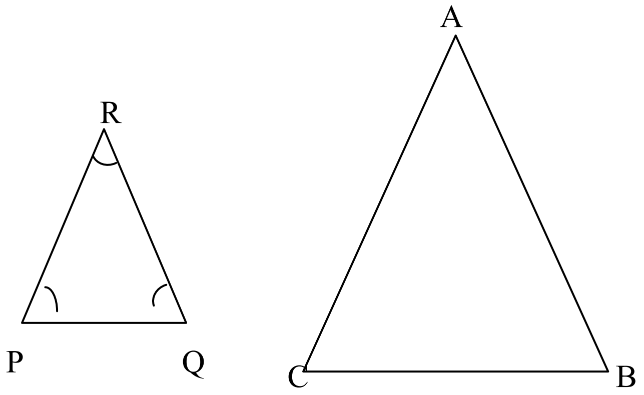


Similar triangles

Two triangles ABC and PQR are similar if in the triangle ABC and PQR

1. $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$, $\hat{C} = \hat{R}$ and

2. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = k$, where *k is a constant.*

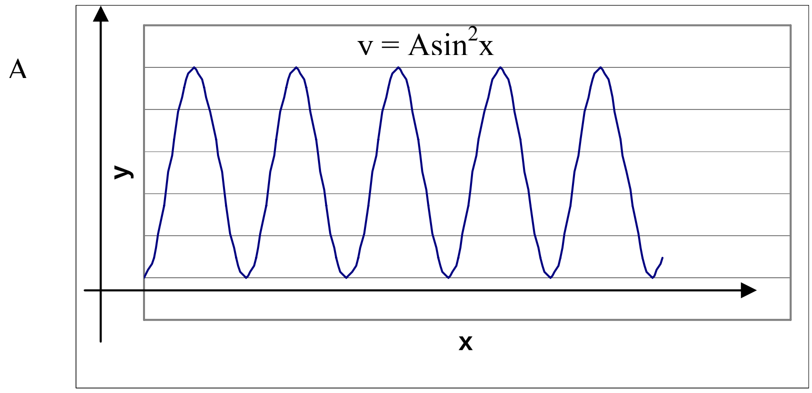


Additional Mathematics, School Certificate

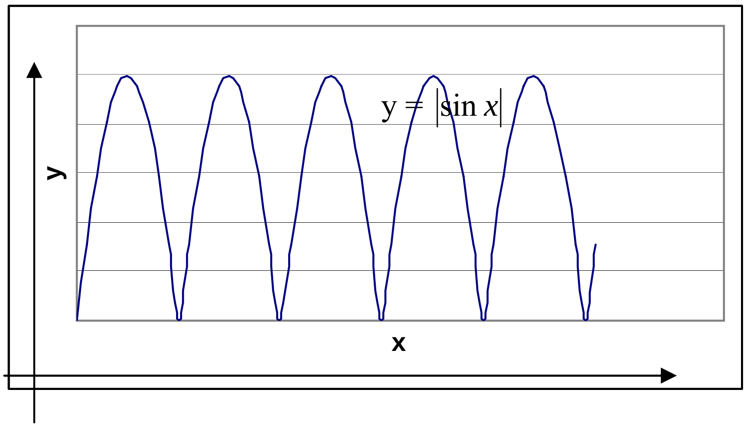
The graph of $y = \sin x$

The graph of $y = A\sin^2 x$ is periodic and it repeats at regular interval of 2π.

However, the graph oscillates between $y = 0$ and $y = A$. It is sketched below.



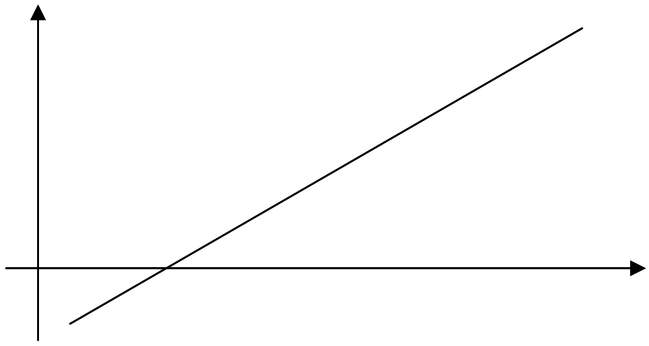
The graph $y = |\sin x|$ is obtained from $y = \sin x$ by reflecting the negative parts of the graph $y = \sin x$ about the x axis.



$y = kx^2$ graph

$y = mx$ graph

$y = mx - c$ is a straight line with gradient m and y intercept $-c$.



Chapter 2

Methodology

The Methodology pertaining to the research study is composed of eight phases:

(i) Research Design (mixture of qualitative and quantitative)

(ii) Critical Analysis of the 'A' physics curriculum,

(iii) Critical Analysis of the Mathematics (Form I – VI) and Additional Mathematics (Form IV-V),

(iv) Testing of prior knowledge and development of questionnaire

(v) Observation of physics lessons

(vi) Development of lessons based on an integrated approach involving core constructs,

(vii) Implementation of the new methodology,

(viii) Evaluation of the lessons through questionnaires

Ten colleges were selected with at least one per region as established by the Ministry of Education and Scientific Research. Six State Secondary Schools and four private schools have been selected. The colleges are named C1-C12 to respect anonymity as per the code of ethics in research.
Formal access was formally sought from the Ministry of Education and Scientific Research and from Private Schools as well.

The study was also conducted in one college in Rodrigues based on the above methodology. Initially two colleges were selected, but one has to be dropped out since it has no physics classes at Form VI. Three members of the team proceeded to Rodrigues to conduct the study.

Research Design

The first step that needs to be taken into consideration while embarking on the research study is to decide on the type of research, that is, should it be qualitative or quantitative? After discussions, it has been decided to add some ingredient of quantitative aspect to the qualitative one. There is a need to point out that the mixed mode enables the researcher to "employ strategies of inquiry that involve collecting data either simultaneously or sequentially to best understand research problems" (Creswell, 2003, p18). Though mixed mode of research is relatively new, we have chosen it due to the following reasons:

(i) Human nature and behaviour are variables that do not remain constant as a result of the essence of evolution and any prediction on cognisance in time cannot be certified by any means since there exists the matter of individuality (Csikszentmihalyi, 1997). – Qualitative data

(ii) Trends can be obtained from marks that will enable us to generalise informed actions. – Quantitative data

(iii) It enables internal triangulation to be carried out, thus cross validation of data as well as corroboration of data is possible (Creswell, 2003).

The Qualitative Paradigm

Strengths

Miles and Huberman (1994) consider that qualitative approaches can deal with "naturally occurring, ordinary events in natural settings" (p. 10). The researcher can therefore gain access to real life situations and eventually do more justice to the subjects. To note that in the qualitative approach, the researcher uses his/her own understanding of events in the analysis of social settings, and this makes his/her understanding of human life and the natural world more meaningful. A purely quantitative analysis would not have allowed us to obtain rich data on the behaviour of learners. It would have simply provided a normative trend which would not tell us about the problem the learners were in fact experiencing in the learning of physics.

Limitations

The researchers being the source of data collector were fully aware of the element of subjectivity. Much care and effort were taken to minimise such intrusion in the research. But, needless to say the own values, beliefs and perceptions assisted the tacit understanding obtained from the research. The fact that more than one researcher was involved in data collection and analysis, an objective stance was more likely to happen after discussion. To note that a friendly relationship was established with the students and this enabled the researchers to collect a wide range of data.

The Quantitative Paradigm

This research study includes a quantitative section whereby the test for prior knowledge was analysed using the SPSS package.

The Mixed Paradigm

"Combining methods may be done for supplementary, complementary, informational, developmental, and other reasons", (Strauss & Corbin, 1998, p.28). The essence of using both approaches has also been highlighted by Breitmayer, Ayers, and Knafl, (1993). "Each adds something essential to the ultimate findings, even to the final theory..." (cited in Strauss & Corbin, 1998, p.28). There can be back-and-forth interplay between combinations of both types of procedures, with qualitative data affecting quantitative analyses and vice versa. The issue in this study was mainly concerned with how the combination of methods could work together to foster the development of theory.

Furthermore, Fraser (1991, 1994) and Fraser & Tobin, (1991) (cited in Suarez et al., 1998) recommends a combination of qualitative and quantitative methods when investigating in classroom environment. This way of proceeding adds richness to data collection and improves triangulation thus bringing credibility to the results.

Ethics in Research

Since qualitative research is usually interpretative research, it is imperative to seek the approval of 'gatekeepers' to gain access to schools and to the participants, as it is a matter of fact that the researchers may bring to the surface elements of confidentiality or past experiences. Cohen and Manion (1994) consider that

> Social researchers must take into account the effects of the research on participants, and act in such a way as to preserve their dignity as human beings (p. 359).

Consent from the Ministry of Education and Scientific Research as well as Private Schools were sought for schools where the study was to take place. The participants as well as the teachers were briefed about the following:

- The research objectives,
- The types of data to be collected,
- Participants' interest will be considered,
- Opportunities for withdrawn from the study,
- All responses will be anonymous.

The 'A' level Physics Curriculum and Mathematics Curriculum

Prior to setting up the ball rolling, it was very important for the research team to identify the various concepts students encounter at 'A' level physics. In this perspective an extensive analysis, longitudinal and cross-sectional, has been made. This exercise enabled the research team to identify and categorise the physics concepts in levels of difficulty. For example,

Concepts	Low level	High level
Mass	✓	
Weight	✓	
Force		✓
Pressure		✓
Acceleration of free fall	✓	
Terminal velocity		✓
Projectile motion		✓
Momentum		✓
Conservation of momentum		✓
Newton's Law of Gravitation		✓
Potential at a point		✓
Kinetic energy	✓	

A detailed analysis of the individual concepts at 'A' level is included in the literature review.

During the analysis phase, it became inevitable that we perform a similar analysis with the mathematics curriculum to identify which are those mathematics concepts that are directly linked with the physics or which mathematics concepts will be needed for students to develop understanding physics. The mathematics Form III-V curriculum was targeted. This productive exercise has enabled us to **develop the strategies pertaining to integrative knowledge and core constructs.**

We are using the term 'core constructs' to describe the process of coherence and association related to the process of schematic learning.

The analysis of the physics and mathematics curricula is presented in the literature review.

Testing of Prior Knowledge

Human development is principally based on pre existing knowledge. Lake (2004) is of the view that "the linkages that students make between their prior knowledge and their current investigation can reveal the nature of their thinking", (p. 108). In this perspective, it is necessary that science teachers are aware of the difficulties that students face in understanding some mathematics concepts (Lenton & Stevens, 1999). Moreover, the authors are of the opinion that though learners are able to perform mathematical tasks effectively in mathematics lessons, they are unable to transfer those skills in the science lessons. Schon (1987) brings about a different dimension to pre-existing knowledge; civil engineers should have knowledge of the soil, texture, etc, prior to building roads. In this way, there is continuous refinement of acquired knowledge which eventually leads to the formation of core constructs.

A questionnaire (Annex 1) to test prior knowledge in physics was developed based on physics knowledge of the students at Form IV-V. Some questions require a sound understanding of mathematics where as some need a qualitative interpretation of physics. The responses will enable us to work out a paradigm for the implementation of our philosophy of core constructs and integrative knowledge.

In this study, it was necessary to test prior knowledge of participants studying physics at the H.S.C level.

The main objectives of setting question based on prior knowledge to participants were to determine:

- whether there is evidence of basic conceptual understanding of physics at least up to School Certificate (S.C) level
- whether there is evidence of adequacy in understanding basic knowledge of mathematical reasoning

Procedure for prior knowledge

- A number of themes were selected from the S.C physics syllabus
- Key concepts were then drawn from the themes
- A set of questions were devised using the selected themes and concepts
- 20 questions were selected to form part of the questionnaire
- Each question was formulated using

(i) appropriate form and structure

(ii) appropriate use of concepts with respect to the selected themes

- There are more than one correct answer in the following questions:
- The participants will be informed that they should expect more than one answer among the 20 questions that are contained in the questionnaire. The main purpose is to make sure that the participants will read all the list of possible answers before attempting the set of questions.

How to administer the questionnaires for prior knowledge

Prior to administrating the set of 20 questions to participants, a pilot pre-test was carried out with a sample of 60 lower VI students from three different secondary schools not involved in the research. The questionnaires were then slightly modified and then administered to participants. (C1 -C10).

Questionnaire for Prior Knowledge

Question No 1: Measurement of length

In this question, five answers have been provided where there are two right answers. Answer 'C' is the distracter as students may associate the term "measuring the length" with "calculating the thickness". It is to be noted that the topic of measurement is covered as from Form II. The concept of measurement has been explicitly covered in that chapter. Students usually do not pay attention to the aspect of comparison when making a measurement.

Question No 2: Interpretation of graph

In this question, six answers have been provided where two only are correct. Our experience has shown that students do not think logically in attempting similar questions. Students usually associate rest position with a horizontal line on a graph. Here, students need to apply their mathematics knowledge of differentiation to cross check their prior knowledge. Alternately, students can split the time axis into small intervals and then determine whether there has been any change in distance for each time segment.

Question No 3: Free fall motion

In this question, six answers are provided with two correct one.

When an object is in a state of free fall, it is important to note that acceleration of free fall is constant, provided the fall is close to the earth surface. The question makes mention to neglect air resistance so that students may link their knowledge of terminal velocity where net force is zero with a situation when net force is not equal to zero.

Question No 4 & 5: Displacement – time graph

In question No 4 & 5, there are six answers and there are two correct answer for question No 4 and one correct answer for question No 5 respectively. The graph shows motion in two different directions relative to an origin. Students are expected to apply their mathematical knowledge with respect to differentiation to answer the questions and to avoid any misinterpretation. Students have a tendency to think that if the inclination of the line is upwards, motion is uphill; which is purely a misconception.

Question No 6: Circular Motion

There are five answers with only one correct answer. Students' knowledge of tangent learnt in mathematical should be applied here. There is a tendency to think that once the string breaks, the stone will move towards the centre because of the centripetal force.

Question No 7: Terminal velocity

There are six answers with two correct one. Students need to extensively apply their mathematical knowledge and to determine when the net force equals to zero since in the consideration of terminal velocity, air resistance cannot be neglected.

Question No 8: Density

There are three correct answers and two distracters.

Students need to refer to their pre-existing knowledge on the ratio of mass to volume.

Question No 9 and 10: Newton's 2nd law

There are three correct answers for both questions. Students usually encounter difficulties in bridging the gap between what they learn in mathematics with physics. Here, the vector sum of forces in mathematics is synonym to resultant or net force in physics. ***F*** from ***F*** *=m**a*** does not refer to a single force and that *acceleration* is always in the direction of *force*. If this was not the case then a negative sign would be present in the equation, implying that the *acceleration* is in the opposite direction to *force*. There is a need to point out that there is a linear relationship between *F* and *a*.

Question No 11: Hooke's law

There are three correct answers in this question and two distracters. The mathematical relationship for Hooke's law: F = ke, is linear provided k is constant. Students should have developed understanding that *k* is a constant of proportionality which is a constant for similar type of spring (provided limit of proportionality is not exceeded).

Question 12: Frictional force

There is only one correct answer. Students usually associate frictional force with an opposing force which always acts in opposite direction to direction of motion. In situation where the object is rolling the situation might be different.

Question 13: Refraction

There is only one correct answer. Students who have not developed core constructs might think that during refraction there is always deviation of light. Change in direction of light does not occur when the ray is incident normally.

Question 14: Pressure

There is only one correct answer. Student should invoke a pressure difference that will force the balloon upwards. From the defining equation of pressure, $P = \frac{F}{A}$, it follows that the pressure difference is in the upward direction and therefore the net force should act upwards.

Question 15: Magnetic field

There are two correct answers. Students have encountered the term field since form III and in form IV-V they covered chapters of gravitational field, electric field and magnetic field. They should have developed an integrative approach to knowledge.

Question 16 & 17: Heat

There are two correct answers for question 16 and one correct for question 17.

Question 18: Electric current

There are three correct answers to this question. A wide number of researches have shown that students still experience difficulties in solving problems in electricity.

Question 19: Parallel circuits

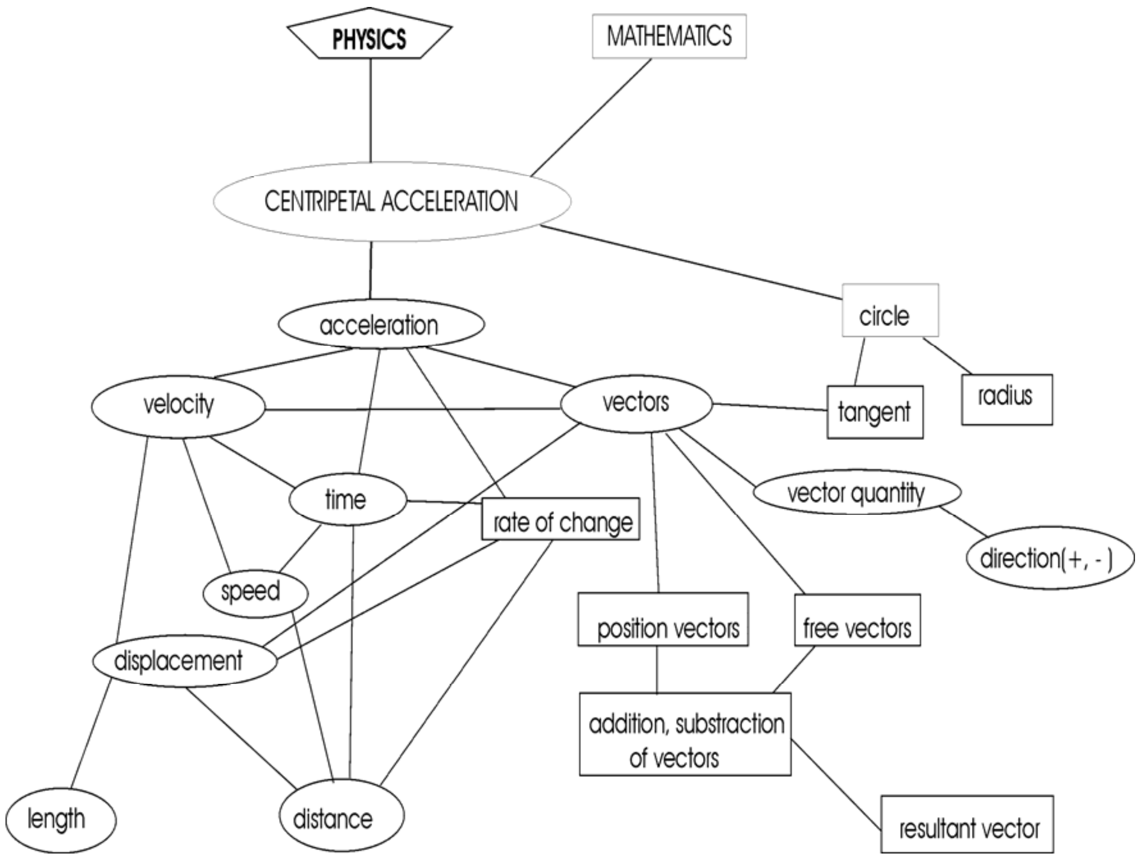
There are two correct answers to this question. Students should have wondered why the batteries in their electrical devices are not connected in parallel.

Question 20: Series and parallel connection of meters

There is one correct answer. This question is aimed at verifying students' ability to recognise whether current will pass in a given branch if there is no resistance.

Identifying Core Constructs

Before the preparation phase of lesson the research team proceeded with the task of identifying core constructs between mathematics and physics of a concept, for example, centripetal acceleration (Parmessur et al., 2002) using a concept map. Concepts maps enable the teacher to organise instructional knowledge with a view to helping students to visualise key concepts and principles together with the interrelatedness between concepts (Novak, 1991).



The concept of centripetal acceleration involves a number of physics and associated mathematics concepts. From the above (Parmessur et al., 2003) concept mapping, the rectangles indicate mathematics concepts while the ellipses indicate physics concepts. However, there is overlapping of mathematics and physics concepts. In order to explain any physics concept it is necessary to work out a concept mapping with the inter-related association of concepts as shown above. Should the teacher explain all the related linkage in order to explain, say, centripetal acceleration? One way to simplify the lesson would be to adopt the questioning technique to challenge pre-existing knowledge of students. Kerry (1986) rightly infers that teachers should always encourage students to use the scientific method and to make sure that they reach “logical conclusions” (p. 43). Challenging one’s view of a particular concept allows pre existing knowledge or misconception to be unveiled. Now, just unveiling misconception is not sufficient, it is important that teachers use students’ shortcomings to devise strategies to enabling them to construct meanings.

Observation of physics classes

Observation plays an important part in any research for it allows the researcher to

> probe deeply and to analyse intensively the multifarious phenomena that constitute the life of the unit with a view to establishing generalisations about the wider population to which that unit belongs (Cohen and Manion, 1994, p. 106-107).

Preliminary observation was carried out to construct a schedule (Annex 2) with a view to maintain consistency among the researches. At least two researches were present during the observation phase. The teachers were clearly briefed on the purpose of the process and assurance was given that all information will be kept anonymous as the schools’ names have been codified.

The observation schedule is based partly on Bloom’s Taxonomy. The schedule comprises of the flowing:

- Aims of the lesson,
- Learning outcomes,
- Adequate planning of the lesson,
- Pre-requisites (testing of),
- Methodology,
    - ✓ Good introduction
    - ✓ Systematic development of lesson,
    - ✓ Good questioning techniques based on:

        - Knowledge
        - Comprehension
        - Application
        - Analysis
        - Synthesis
        - Evaluation
    - ✓ Response time
    - ✓ Linkages with mathematics (teacher's perspectives)
    - ✓ Linkage with mathematics (student's perspectives)
- Conclusion.

Observations were conducted in the following schools: C2, C3, C4, C10 and C11.

Teacher's Questionnaires

On the first visit to schools, an unstructured interview was conducted with a sample of teachers with a view to gathering preliminary data.

Some of the key questions are:

- Do your pupils face difficulties in the learning of physics?
- What kind of problems?
- Why do they encounter these problems?
- How do you help them to solve these problems, if any?
- Do you make use of other subjects to help your pupils understand physics better?
- If yes, which subject(s)?

A questionnaire (Appendix 3) was set to the teachers and this is in conjunction with reliability and validity in educational research. The questionnaire comprised of statements:

- of general nature,
- related to types of questions set by the students,
- referring to whether students are aware of underlying mathematical concepts involved in physics,
- about problems students face in studying physics at ‘A’ level,
- concerning interdepartmental collaboration.

To note that the questionnaire was pilot tested with a sample of 3 teachers not involved in the study. Slight modifications were brought to the final one.

Students’ Questionnaires

With a view to obtaining feedback from students on lessons conducted by the researchers, a student’s questionnaire (Appendix 4) was devised. The questionnaire comprised of statements:

- whether students appreciated the teaching,
- whether the questions were challenging,
- whether the question helped them to better understand the concepts,
- whether they liked the moment that mathematics was linked with physics,
- whether all physics lessons should be taught this way.

Again, the questionnaires were pilot tested and a few amendments were made in relation to language.

Chapter 3

Findings: Prior Knowledge

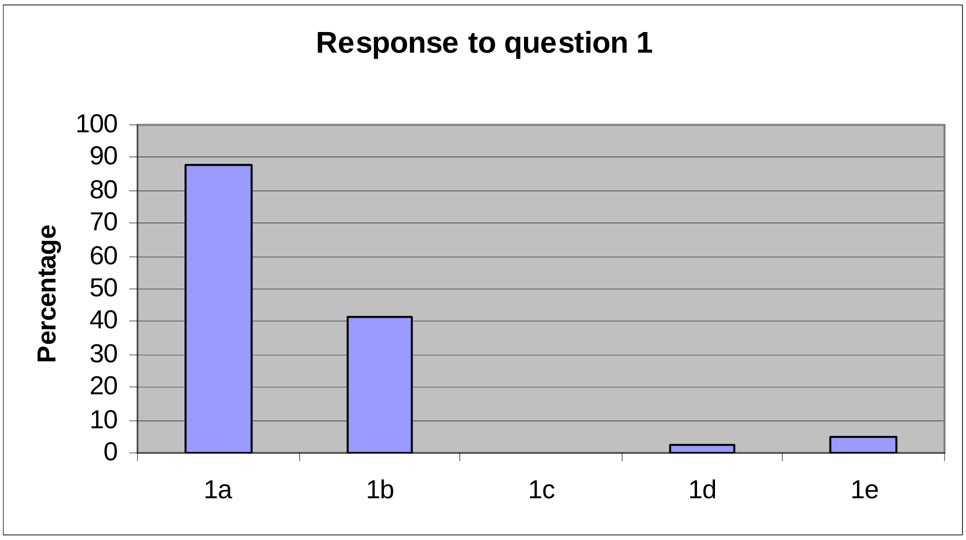
Analysis of response to the questionnaire on pre-requisites knowledge of students.

In the first instance it was found necessary to investigate the pre-requisite knowledge that students possessed in physics as this formed the basis of our research. Trowbridge et al. (2000) are of the view that teachers have the impression that students are like empty vessels or “blank slates” (p. 179) and that they are starting to learn concepts from scratch. We should not forget that students come to the classroom with a load of information, conceptually correct and incorrect. The teaching process should accommodate this fact and redress the situation to find out the level of inconsistencies that students hold.

A sample of schools was chosen in which one Lower Six class was selected. All the students in the class were asked to truly answer the questions found in the questionnaire which was administered to them. There were twenty questions in all, based on the physics Form IV-V syllabus. All the questions were multiple choice types and most of them had more than one correct answer. The purpose of this exercise was to find out to what extent the students have mastered the associated concept in physics and have acquired conceptual understanding in physics and mathematics. In this analysis, the response of the students for each of the possibility is given on a bar chart.

Question No 1: Measurement of length

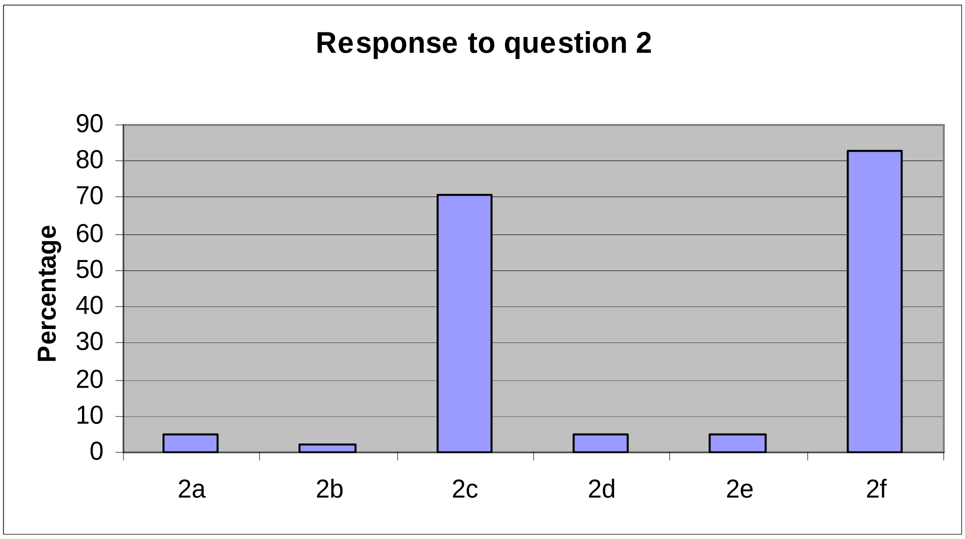
Objective: To investigate into students’ mastery of the fundamental principles of the concept of measurement.



While almost all the students answered correctly “**determining the dimensions of the object**”, less than half of them also identified the other correct answer “**making a comparison against a standard length**” which deals with the basic principle behind measurement. Though measurements (length, mass, capacity) form a part of the basic knowledge in science and mathematics since primary level of education and students have been exposed to activities related to measurements since childhood, many of them have not really grasped what exactly is involved while doing a measurement. The answer to this question demands some thinking in terms of whether students have developed core constructs or they are simply engaged in superficial learning.

Question 2:

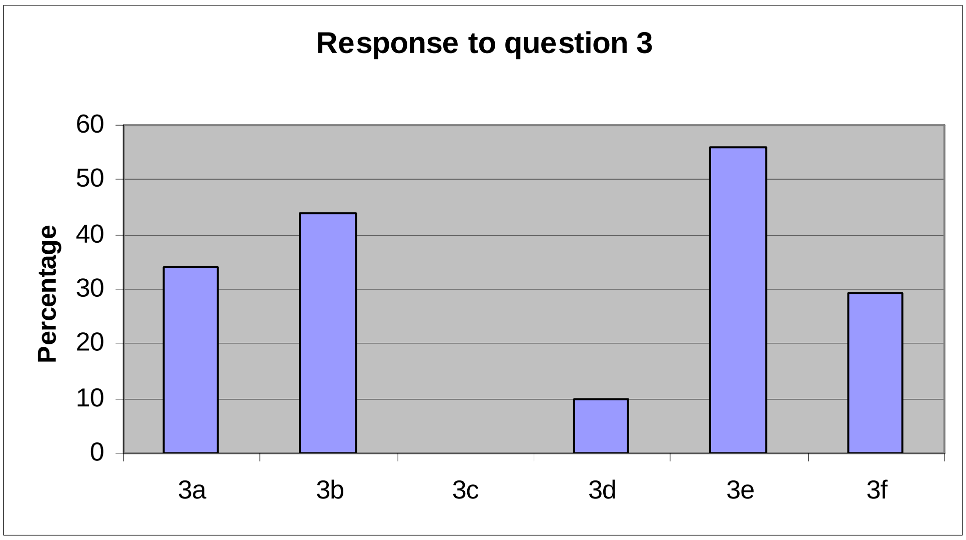
Objective: To find out whether students pay attention to the labels on the axes of graphs.



Many students were able to identify the two correct options which showed that motion has stopped. One of the common problems students encounter is not paying due attention to the labels on the axes and this leads to misinterpretation of the relationship between the two variables. The answers clearly point out that most students have mastered this type of relationship though a few students still encounter difficulties.

Question 3:

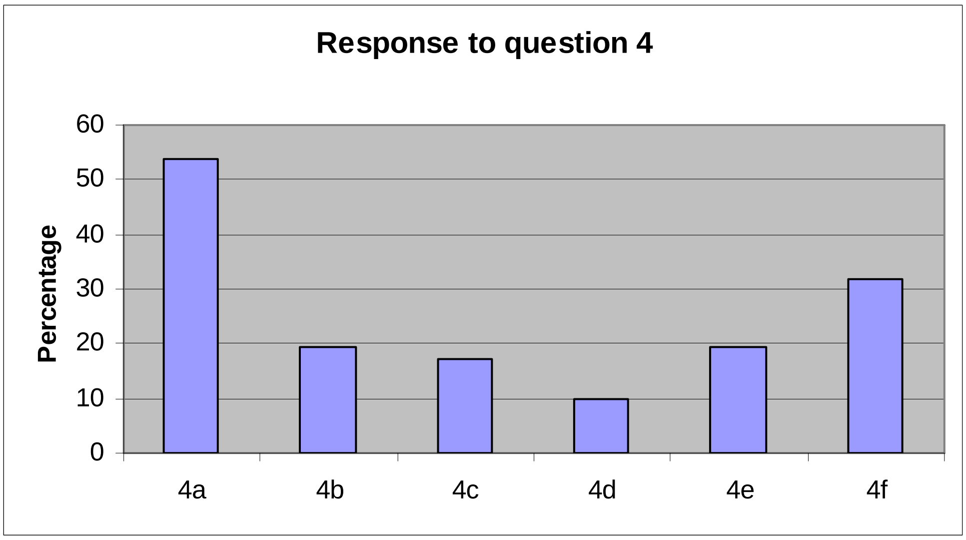
Objective: To determine the conditions under which an object is in a state of free fall.



In this question there has been a variety of answers for what happens when an object is in a state of free fall (neglecting air resistance). While the correct answers have been identified as B and E, other answers have also been mentioned namely **“the velocity remains constant”** and **“the acceleration is zero”.** Such a variety of responses clearly indicates that students still withhold misconception related to forces acting on bodies in a state of free fall. Here, it is worthwhile to note that students have difficulty to consider situation when there is free fall, that is, air resistance is neglected and that the net force is the force due to gravitational influence. The acquisition of scientific knowledge only tends to be fragile, disorganised and therefore poorly constructed. Osborne & Collins (cited in Osborne et al. , 2003) are convinced that “science education is science’s own worst enemy, leaving far too many students with a confused sense of the significance of what they have learned” (p.694).

Question 4:

Objective: To investigate relationship between two variables in a displacement-time graph.



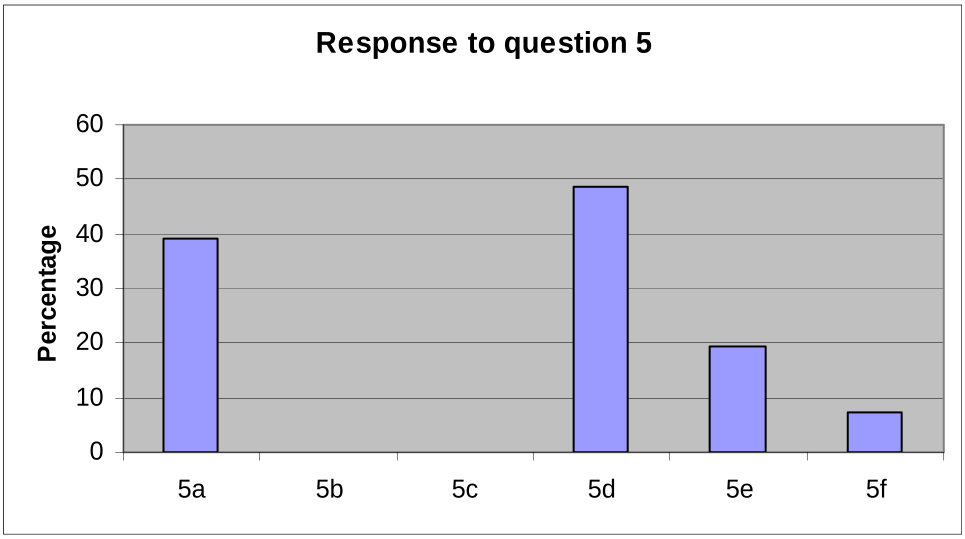
Just above 50% of the students could identify the correct answer **"motion with uniform velocity"** while only 30% identified the other correct response **"linear motion"**. Among the wrong items identified, one can note "**motion with increasing speed**" and "**motion with decreasing speed**". This shows that some students may have anticipated that linear motion is subject to motion in one direction only.

A mathematical understanding of the relationship would have enabled students to cope with the difficulty.

It is a fact that students usually are not engaged in in-depth analysis of the data that they have collected, and that their conclusions are inconsistent with the data (Kanari & Millar, 2004). The authors maintain that one of the reasons could be that students tend to "hold to an initial hypothesis in the face of disconfirming evidence" (p. 760).

Question 5:

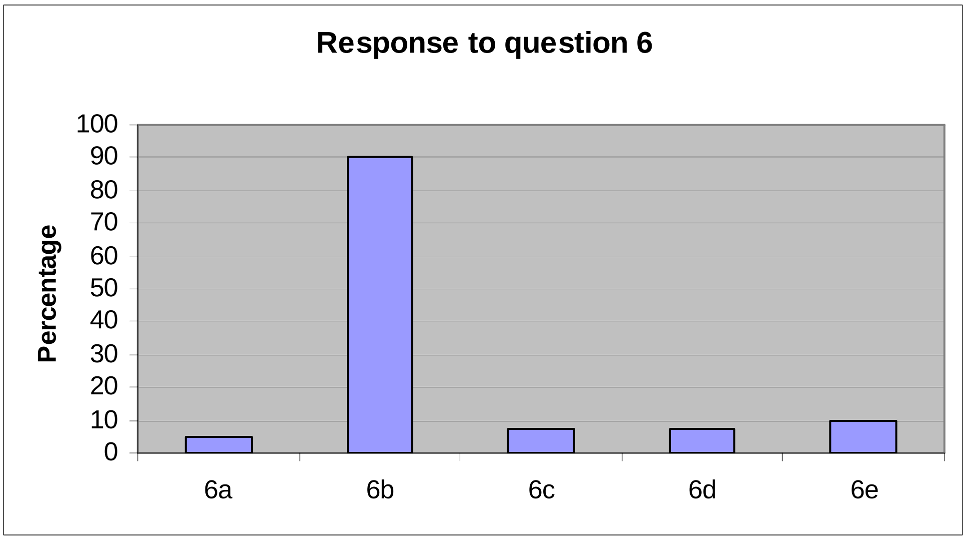
Objective: To investigate further into the displacement-time relationship.



It is surprising to find that many students identified "**motion uphill and downhill**" as one of the possibilities. It seems that they were influenced by the shape of the graph and no proper attention was drawn to the labels on the axes. This question provides further data about the type of misconception students withhold and it provides useful information about the learning process. Dufresne et al. (1995) developed a cognitive model to show the different ways that novices and experts store and use content knowledge. In this example, it is clear that novices have poor clustering of concepts. Ed van den Berg & Grosheide (1997) confirms that students' misconceptions still persist even when they are told about their mistakes and that "a change in context will trigger the misconception again" (p. 89). At times, bridging concepts (analogy) should be used to enable the learners to understand a given principle.

Question 6:

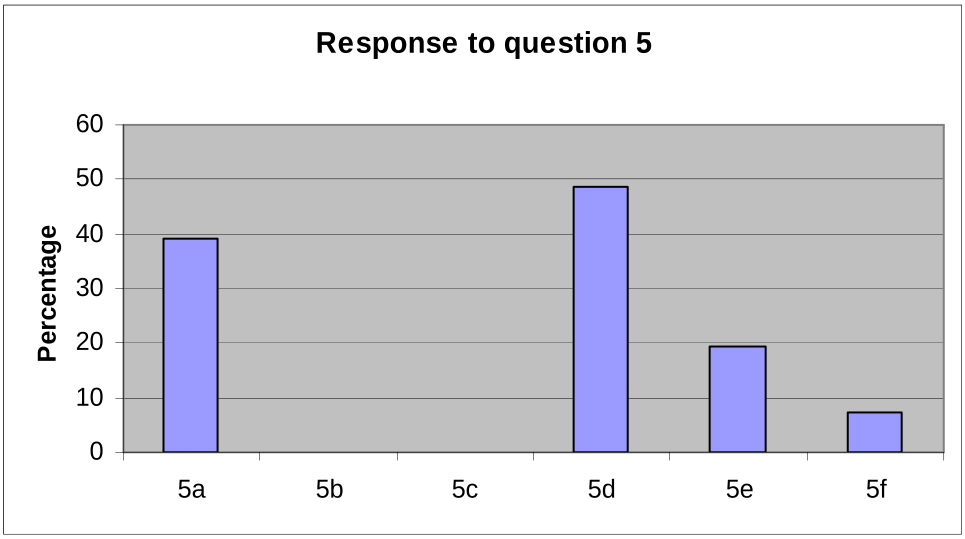
Objective: To determine students' pre existing knowledge of circular motion



90% the students could answer this question correctly, even though a few incorrect responses could be noted. But still the element of misconception is present to some extent. It is possible that to some, learning was superficial and isolated. In mathematics students have been solving numerous problems that involve tangents; if such links have been made, then the formation of core construct would have been favourable.

Question 7:

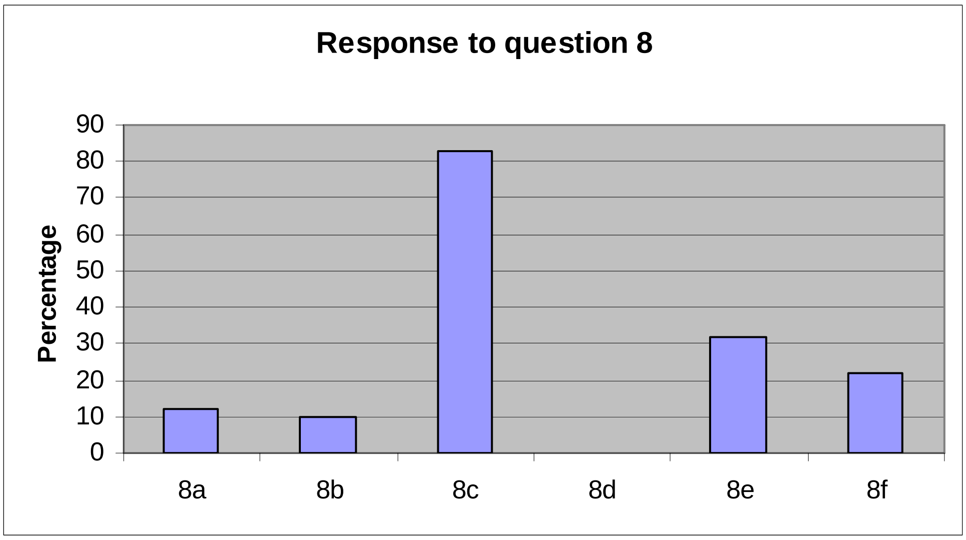
Objective: To determine the state of an object at terminal velocity.



While around 75% of the students could identify the **correct answer "it is falling with constant velocity",** only 60% could identify the other correct one namely **"the resultant forces acting on the object is zero"**. It is worth noting that the concept of terminal velocity could be taught using Newton's second law whereby the students have the opportunity to deduce that the net force is zero. Learners "benefit from teachers who carefully choose *learning strategies* (italics in context) that are demonstrably the most appropriate and effective" (p. 61).

Question 8:

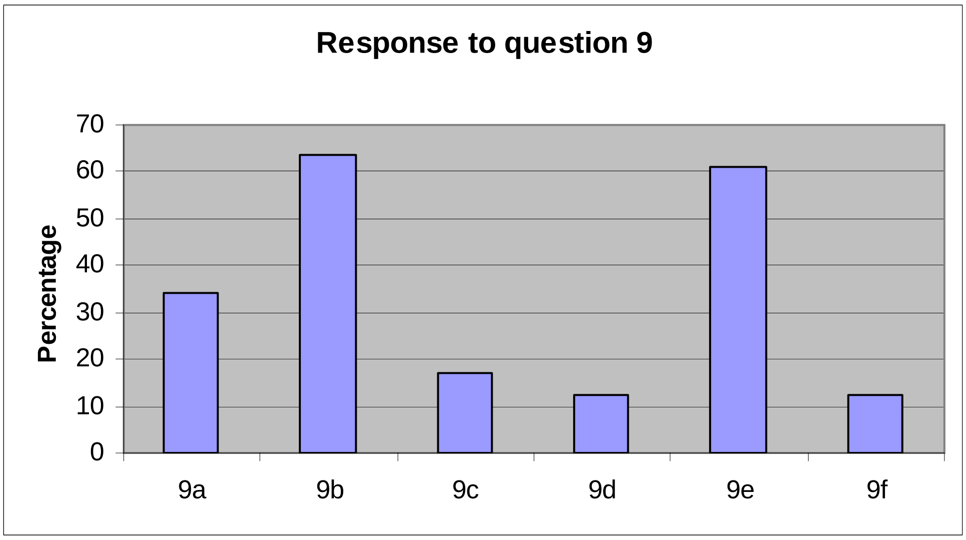
Objective: To investigate the conceptual understanding of students in relation to density.



Out of the three correct possibilities, only the one "**they occupy different volumes**" was identified by the majority of the students. The other two correct ones **"they have different structures"** and **"they have different properties"** were noted by at the most 30% of the number of students. This indicates that students have a very superficial understanding of the concept density in physics, though this has been dealt with since lower secondary level. It is to be noted that density is rarely perceived as the 'compactness' of a substance. The teaching-learning process should be an ongoing transaction between the teacher and the learners. Learners need to be challenged so that their imaginations are triggered with a view to bringing about the development of core constructs. Woolnough (1997) is of the opinion that "there is no limit to what students ... can achieve once they are switched on and self-motivated" (p. 71).

Question 9:

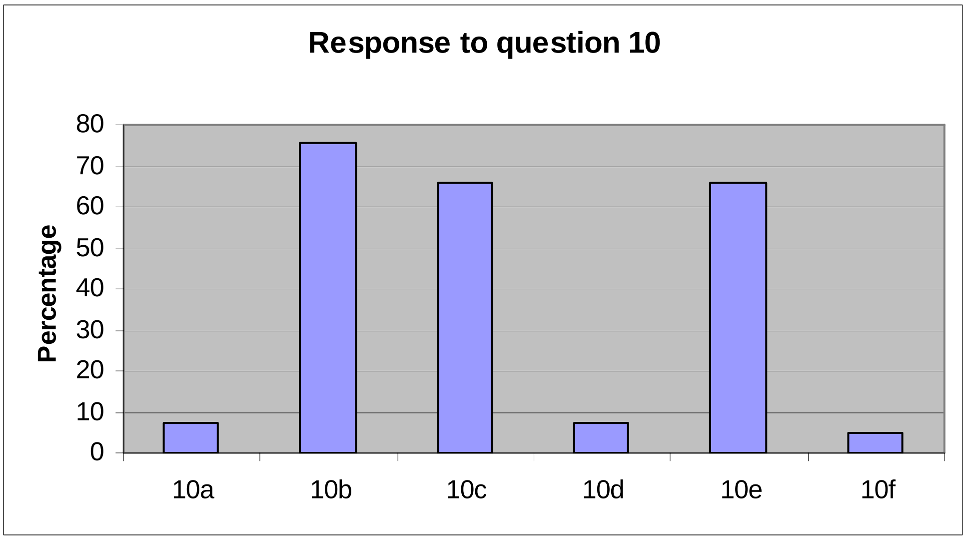
Objective: To investigate the relationship between **F** and **a**, following Newton's 2nd law.



Out of the three correct answers, two of them "**the net force**" and "**the resultant force**" could be identified by around 60% of the students. However, the other correct one "**the vector sum of various forces**" could be identified very rarely and the wrong one "**a single force**" was identified in around 35% of the cases. To note that the 'vector sum' is derived from mathematical arguments and it appears that students have developed separate clusters of mathematics and physics related to the same concept. This is a serious concern for the self development of the learner as physics and mathematics form the foundation courses of engineering and technological subjects (Woolnough, 1997).

Question 10:

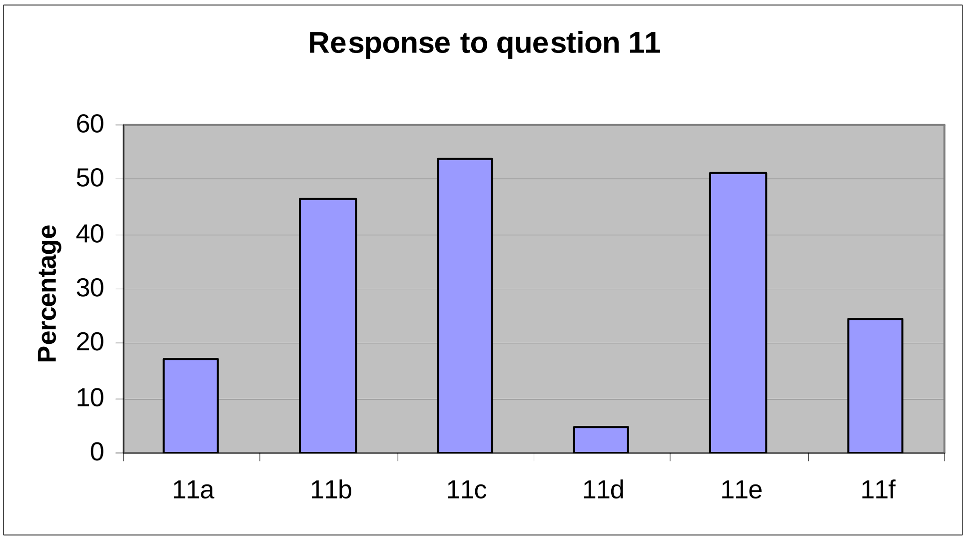
Objective: To investigate further the fundamental equation **F** = m**a**



All the three correct possibilities namely **“acceleration acts along the direction of the applied force”, “the product of a scalar and a vector quantity results in a vector quantity”** and **“the equation is homogeneous”** were identified in most of the cases. It is quite pleasing to note that students have been able to apply their mathematics knowledge to answer the question.

Question 11:

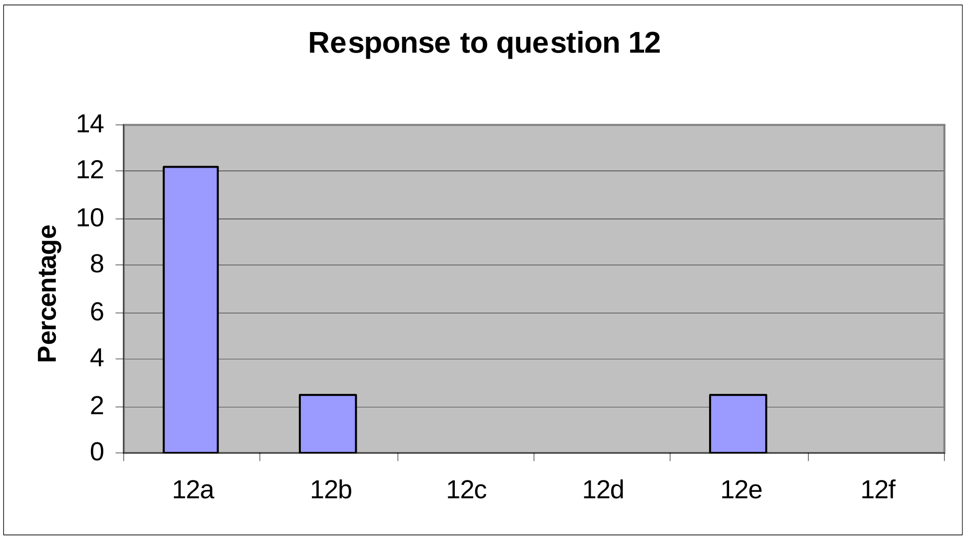
Objective: To demonstrate understanding of spring constant in Hooke’s law.



One can note that out of the three correct responses, two of them namely "**the same value for the same type of springs**" and "**the same value for the same length of similar type of springs**" were identified by the majority of the students. However, the other correct answer "**the same value for any length of similar type of strings**" could be identified by less than 50% of the respondents. It should be noted that the possibility "**different value for all strings**" have wrongly been identified by a vast majority of the students. To be able to understand statement B, students need to invoke critical thinking and analyse the various possibilities that are involved. It is to be noted that "science education without argument is like a book without a plot; in danger of becoming a tale told by an idiot, full of sound and fury but ultimately incomprehensible" (Osborne et al., 2001, p. 69).

Question 12:

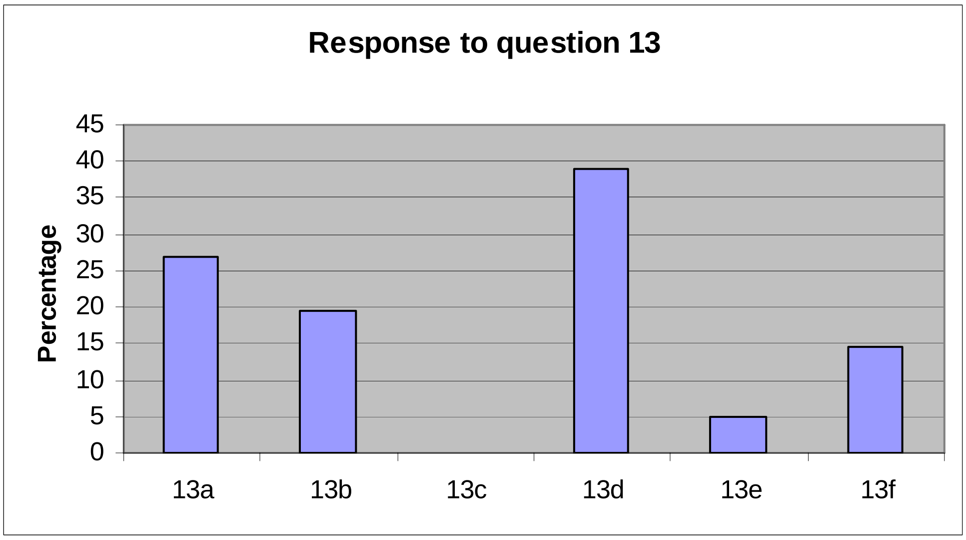
Objective: To investigate frictional force in a real context.



This has been a question which has been answered by very few students and no correct answer has been noted. One of the incorrect responses was "**the direction 1**" which perpendicular to the road; this is totally misleading. It should be noted that in general students do solve problems involving friction quite well; a real conceptual understanding of the concept of friction is not that common. This is a clear example of students' possession of **scientific knowledge** as opposed to **scientific understanding**. Scientific knowledge is learned by rote (Abdullah & Scaife, 1997).

Question 13:

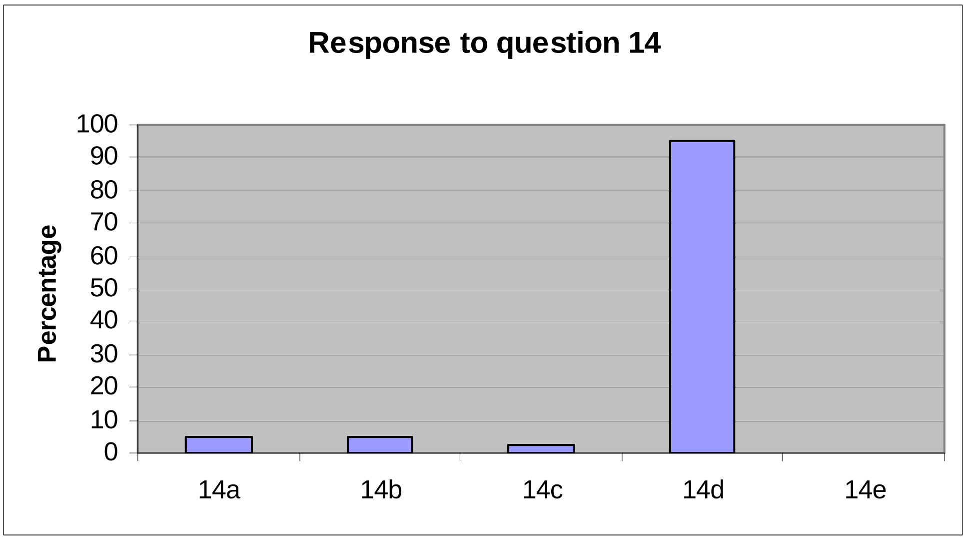
Objective: To investigate the conceptual understanding of refraction.



While the majority of the respondents have identified the correct response "the **ray passes through without being deviated**", more than 50% of respondents also identified the possibility "**the ray is refracted towards the normal".** Students encounter refraction since form III and it is quite surprising that less 50% correctly answered this statement. It is most probably that learners possessed scientific knowledge only. If they have mastered scientific understanding then the problem ought not to have cropped.

Question 14:

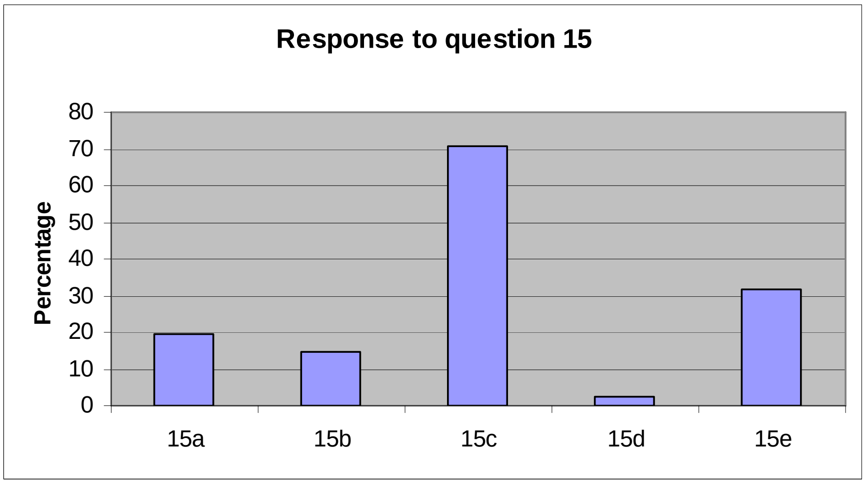
Objective: To investigate students' understanding of pressure difference.



This is one of the questions which have been answered successfully by almost all respondents. A very small fraction considered answer A, B and C to be correct. The answer '**the balloon remains undisturbed and its size increases"** is totally wrong and it is noted that still some students (about 5%) considered it to be one of the correct answers. Again this answer provides evidence of students' non-mastery of scientific understanding.

Question 15:

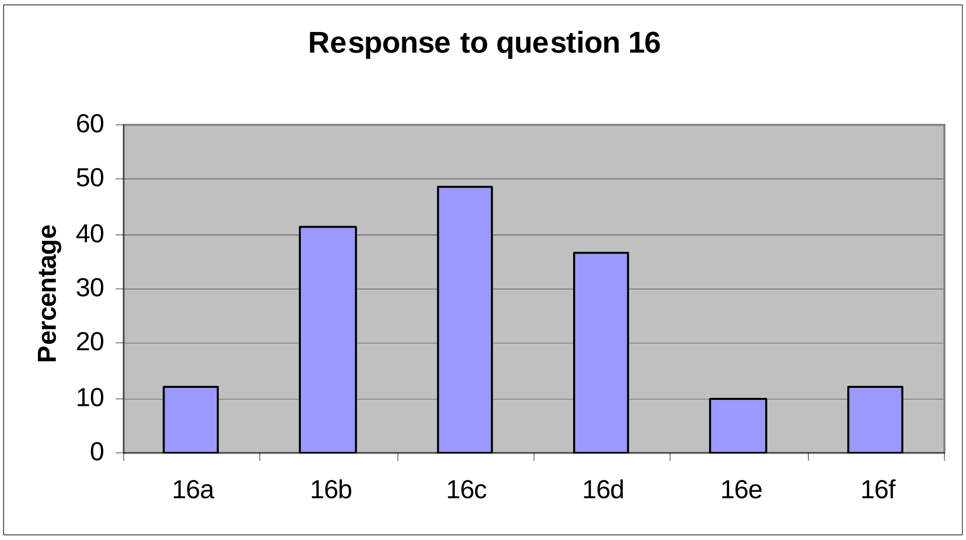
Objective: To investigate students' appreciation of the concept of magnetic field.



While about 70% of the students could identify one of the correct responses namely "**magnitude and direction of the field**", only one third of the respondents could also identify the other correct possibility "**strength and orientation of the field**". In this case, one can anticipate a language problem which is another burning issue in science education. Language problems in science teaching/learning has been thoroughly investigated (for eg. Moje et al., 2001; Cummins, 1984) and this problem is an acute one in countries where English is not the mother's tongue.

Question 16:

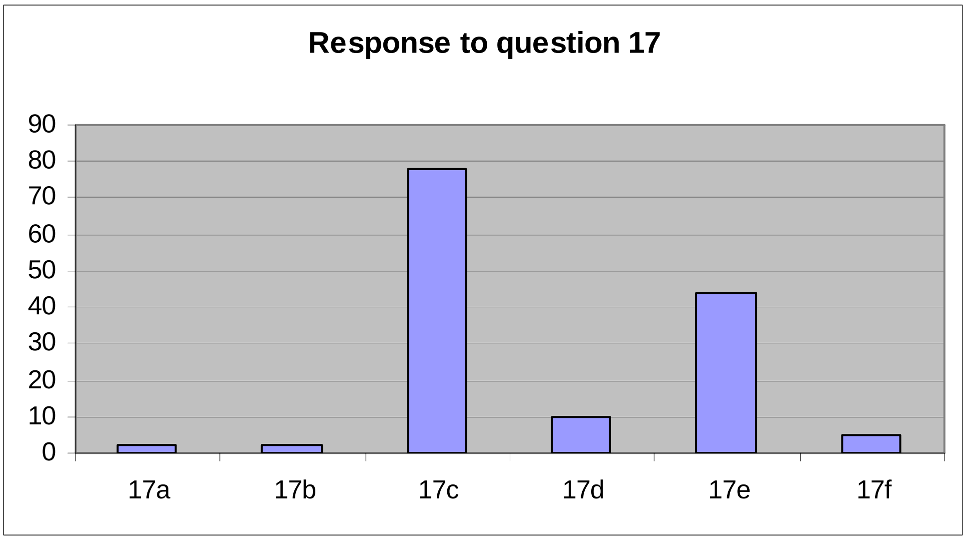
Objective: To investigate the properties of air regarding its use as an insulator in special circumstances.



One of the correct answers namely "**air will transfer heat by convection unless trapped**" has been identified by about 50 % of the students while the other correct one "**air is a bad conductor of heat when trapped**" was identified by about 40% of the respondents. It should be noted that out of the incorrect possibilities identified most often is that "**air molecules will vibrate and transfer the heat unless trapped**". Here, it should be noted that the transfer of heat in non-conductors does not involve vibration which is one of the processes of heat transfer in solids.

Question 17:

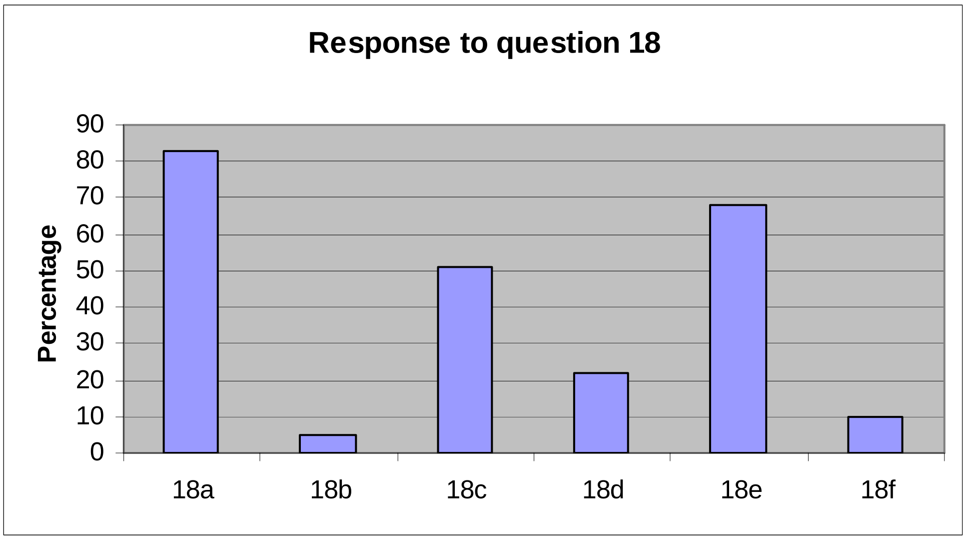
Objective: To investigate how far students have developed mastery of the concept of conduction.



While almost all the students could identify the correct response "**the spoon will conduct heat away from the glass**", around 45% of the students also identified the incorrect possibility "**the spoon will transfer the heat slowly to the glass**". This answer is totally wrong as it bears no logical interpretation since a metal spoon is a good conductor of heat. The answer makes reference to the fact that it is the spoon which acts as an intermediary medium that transfers the heat from the water to the glass at a slower rate. If students considered that this is one of the correct answer, it simply implies that students have not developed conceptual understanding of the concept of conduction.

Question 18:

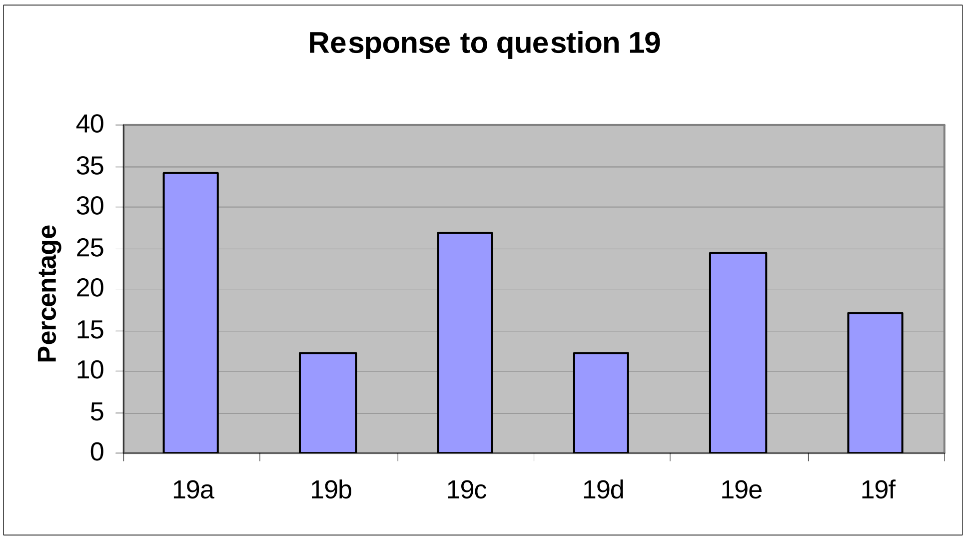
Objective: To investigate into students' difficulties related to current flow.



Out of the three correct responses, the one "**the circuit is closed**" has been identified by most of them. Though this statement is a low-ordered one, it is surprising that around 18% have not been able to identify it. It should however be mentioned that the correct response "**there is a potential difference between the two points**" has been identified by around 70% of the students while the third correct answer "**there is a cell in the closed circuit"** has been identified by about 50 %.

Question 19:

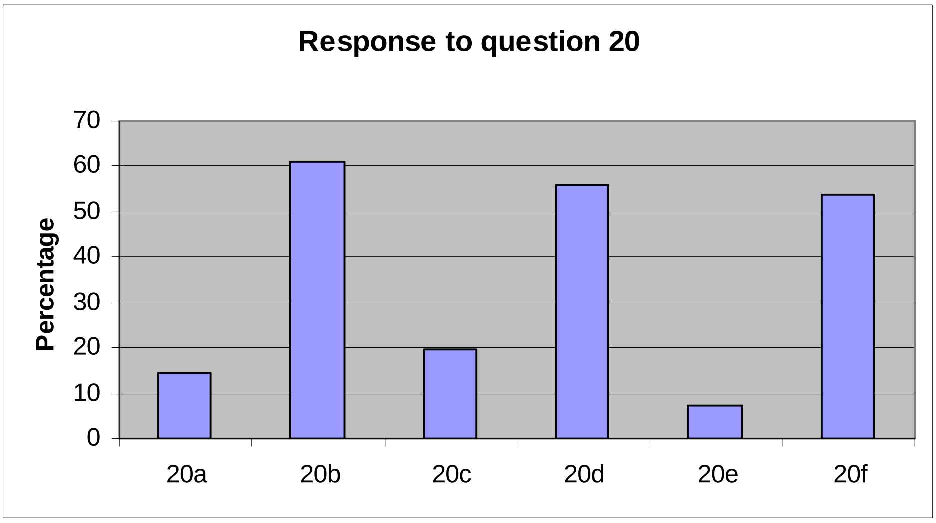
Objective: To investigate students' understanding of parallel circuits.



It is worth noting that this question has posed quite some difficulties to the students as illustrated by the low percentage of correct answers. 25% of the respondents could identify the correct answer "**the current will still flow even if the switch is open**" while less than 15% could identify the other correct answer which is "the **cells will leak**". Out of the incorrect responses noted, one can find the answer "**the emf will be too low"** in about 35% of the cases.

Question 20:

Objective: To investigate into students' understanding of meter connection in circuits.



While the response "**6**" was correctly identified by about 55% of the respondents, it is worthwhile noting that this question has unveiled a number of misconceptions as well as probably misreading of the statement as illustrated by the high percentages of responses "**2**' and "**4**" (60% and 55% respectively). In this question, one can determine that the wire connected across the resistance acts as a short circuit, making the voltmeter at position '6' useless.

Chapter 4

Observation of physics lessons

Before embarking in the teaching phase, a few physics lessons were first observed to actually determine to what extent the teaching strategies invoke critical thinking in students which is responsible for meaningful learning. The main aim of this exercise was to determine the teaching strategies teachers normally adopt, the type of interactions taking place in the classroom and the extent to which students were involved in their own learning. An observation schedule, partly based on Bloom's taxonomy, was designed for that purpose (see Annex 2) and the observations were carried out by all the members of the research team. After each observation session, members of the research team had discussions among themselves to compare and discuss their observations, thus ensuring reliability and validity.

A prototype observation session is described in detail to illustrate the classroom transactions.

The lesson was on Electricity at the lower Six level in school C3. At the beginning of the lesson, just the topic was mentioned to the students without properly stating what were the learning outcomes of that lesson. As part of this topic has already been dealt with at the School Certificate level, the teacher started by asking questions to pupils related to prior knowledge. There was evidence of preliminary planning on the part of the teacher but no demonstration or activity was carried out (though a battery and a bulb were lying idle on the teacher's desk). One thing which was very common in the initial part of the lesson was chorus answers from the students once the question was set by the teacher. It could be anticipated that not all the students were given the opportunity to answer any set questions. The teacher should have given enough time for targeted students to come up with an answer after an appropriate ‘wait time’.

One could also note that the questions set by the teacher were either of knowledge or comprehension level (low order In Bloom’s taxonomy of educational objectives). Such type of interaction will surely encourage rote learning and does not invoke critical thinking. Meaningful learning and rote learning are at the extreme ends of an endless continuum and that learners need to relate the concepts and prepositions to relevant knowledge already existing in learners’ cognitive structures (Novak, 1998).

An example of the lesson is henceforth provided:

Teacher: Can anyone define electric current?

After a short wait time, the teacher provided the answer himself as:

$$I = \frac{dQ}{dt}$$

Our comment: *There is a need to point out that this equation need further probing as it involves the concept of rate which can either be uniform or non-uniform which has not been done during the teaching*.

Teacher: For a steady current, $I = \frac{Q}{t}$

Our comment: *No further explanation on why this should be so was provided. The notion of ‘steady’ was not developed. No mathematical linkage was made.*

Teacher: Can anyone define an ampere?

Student: Ampere is the ratio of coulomb to time.

This response from the student was ignored by the teacher. The definition of ampere was then provided by the teacher himself. In the same way the definitions of Coulomb and potential difference were given as well as their SI units. This clearly indicates the teacher centered approach.

Stereotyped numerical questions such as, *calculate the current flowing in a wire when the charge of 10 C flows for 5 s, were* then set which did not involve problem solving strategies as this would have helped in identifying any misconception.

Another problem was then set involving flow of current in a wire.
Discussion with students followed to deduce that there was a heating effect. The following derivation was then done by the teacher:

$P = I\,V$

$V = I\,R$

Therefore, $P = I^2\,R$

$P = V^2 / R$

No proper opportunities were given to the students to critically think and derive the equations based on prompts.
There were some instances where students did ask questions and it is worth noting an interesting point:
"Why should the wires be of infinite length?"
Our comment: This question could have provided the teacher with an opportunity to engage the students into critical analysis which would have led them to eventually come up to understand the concept.
In short, one can say that the lesson was very much teacher-centered where the teacher was providing most of the definitions and explanations and no proper opportunities for students to reflect were given. When questions were set to the students, at times their responses were not treated and exploited enough to initiate logical thinking and identify any misconception present. There is a need to point out that the teaching was directed at the product rather than the process which would have involved the active participation of the learners. Even questions set by the pupils who could have given rise to much fruitful discussion were not appropriately dealt with. The traditional mode

of instruction that was adopted led the students to rote learning which was perceived by them as being the right thing.
Classroom observations were carried out in four classes in different schools and similar conclusions were reached; that is the traditional stereotyped mode of teaching was the general practice.

It was necessary to probe further into the classroom transactions and consequently unstructured interviews were conducted with the teachers. The unstructured interviews were chosen because it enabled the researcher to obtain rich data since the interviewees were given the possibility to express freely on issues related to the topic of discussion (Cohen & Manion, 2000).

Unstructured Interview

From the unstructured interviews, the teachers claimed that:

- *they are aware that students encounter mathematical difficulties during physics lessons,*
- *they take into account the mathematical problems during the development of the physics lessons,*
- *students encounter difficulties in physics due to their inability to understand concepts that have already been explained (prior knowledge),*
- *they make in-depth analysis of integration or differentiation during physics lessons,*
- *they ask lots of questions to students,*
- *they find language not to be problem in the teaching of physics, but at times they have to resort to Creole.*

However, data obtained from the observation phase revealed a completely different picture.

After classroom observation sessions, lessons were conducted by one of the researchers using an approach which enabled students to logically link mathematical concepts with those of physics. The aim of this phase of the study was to find out whether through the active participation of students, the link between logico-mathematical concepts and physics led to:

(i) realize the importance of mathematical concepts in learning physics
(ii) enhance their conceptual understanding of physics.

The lessons have been devised in conjunction with a model of thinking which the researchers have developed during the course of this study. Furthermore, the evaluation procedures adopted during the physics lessons were based on a new model of assessment and evaluation developed by the researchers.

In chapter 5, the model of thinking together with the comprehensive interactive process model of evaluation are outlined.

Chapter 5

Theoretical Framework

A Thinking Process Model

This research study would have not been possible to carry out without a thinking paradigm. Before the implementation phase in applying core constructs in the teaching/learning of physics, the team members elaborated a thinking paradigm which encompasses some levels of thinking. Thinking is referred to “a set of processes whereby people assemble use and revise internal symbolic models” (Gilhooly, 1996, p.1).

Leonard et al. (1999) differentiated between expert and novice knowledge where the domain knowledge has been divided into three categories; operational and procedural knowledge, problem state knowledge and conceptual knowledge. In this model, the authors have developed hierarchical clustering of conceptual knowledge whereas in novices, there is poor clustering of concepts. Therefore, there are weak or non-existent links between operational and procedural knowledge with conceptual knowledge unlike the case for experts where the links are bi-directional.

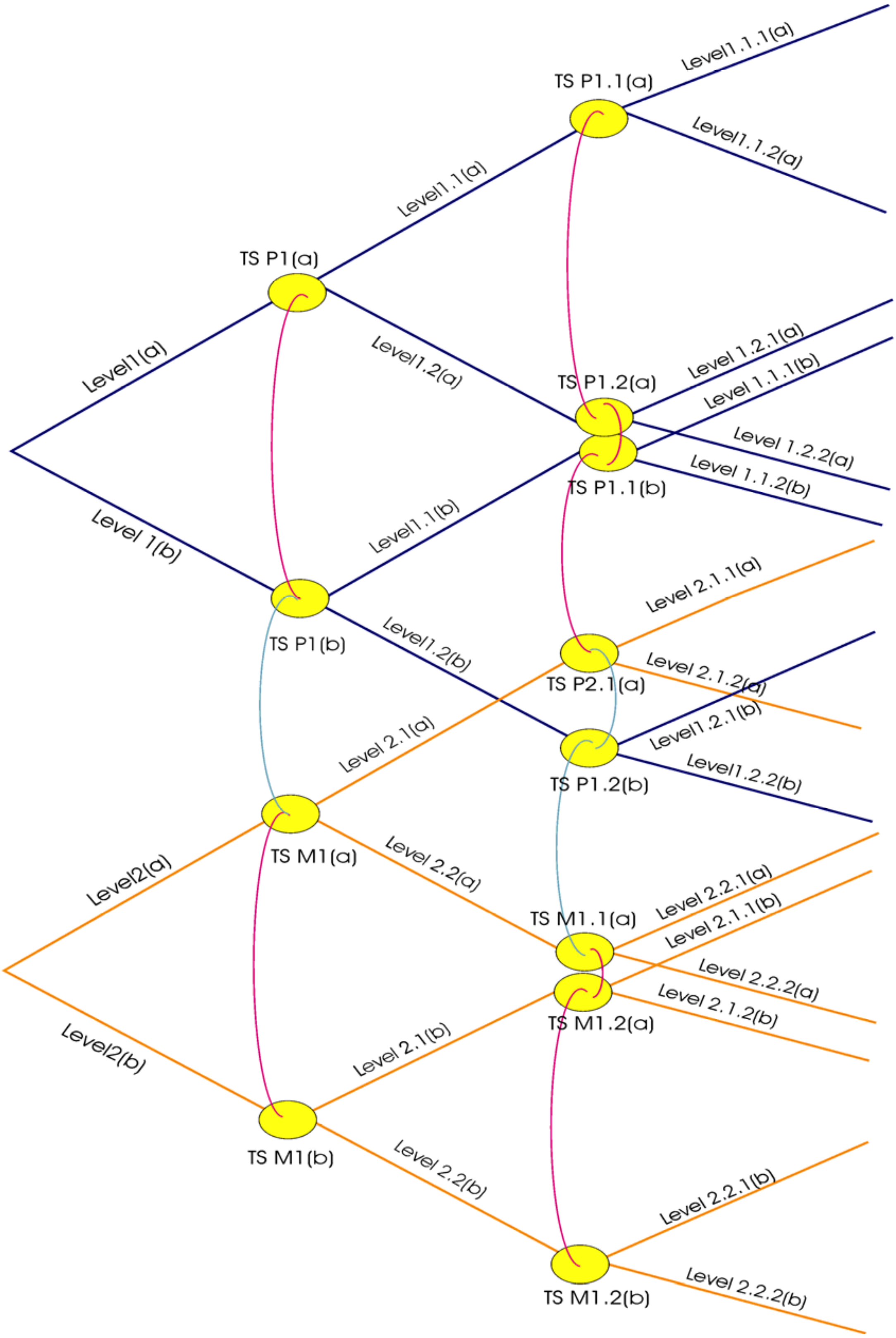
Stillings et al. (1995) proposed a model for cognitive processes, working memory and attention. In this Model, schema forms the basis of any thinking process. A schema is “any cognitive structure that specifies the general properties of a type of object or event” (p. 33). Schemata form prepositional networks where organised structures are processed.

In order to fulfil the objective of the research study, it was imperative to set a framework of thinking processes that would enable us to determine the state of knowledge possessed by the learners. In this context our group have developed a cognitive model of operation within core constructs that helped us analyse conceptual understanding of learners. This Model is one representation of knowledge structures and it needs to be constantly modified to accommodate new development in thinking processes.

The Model has enabled us to:

(i) analyse and challenge pre-existing knowledge,
(ii) investigate conceptual difficulties of students,
(iii) determine the level of thinking and understanding in physics and mathematics,
(iv) enable the learners to develop core constructs.

The Thinking Process Model



In this model, the thinking process is depicted in hierarchical levels; the levels are at times dependent on one another or at times independent of each other. The diagram illustrates the thinking processes for physics (blue) and mathematics (orange) as well as their interrelationships. The ellipses joining the different levels are a platform that the learner can use to jump from one level to the other. However, such process is an effortful one and demands logical thinking to some extent. The teacher has to ensure that learners are using a wide repertoire of cognitive structures. Adequate training is required to be able to jump from one level to another. **The gaps represent instances or transition stages when core constructs are developed whereby critical thinking as well as lateral thinking promote such development.**

In order to reach a certain level of thinking, one has to undergo a logical sequence of thinking, for e.g. to reach a given level of thinking, say, level 1.1 (b), it is necessary to proceed along any of the following thinking 'routes':

(i) L1(b) -{TS P1(b)} - L1.1 (b)

This route is considered as involving low order thinking process, being linear it requires less effort, though it involves the development of core constructs, but at a lower level of thinking.

(ii) L1(b) - {TS P1(b)} - L1.1(b) ↔ L1.2(b),

Here also, low order core constructs should have been developed with a possibility of developing lateral thinking (represented by the symbol ↔).

(iii) L 1(a) - {TS P1(a)} - L 1.1 (a) ↔ L 1.2(a) –{TS P1.2(a), TS p1.1(b)} - L1.1(b),

This route is more complex than in (i) and (ii) as it involves crossing over a number of gaps, referred to as '*transition stage "TS"*'. It is an effortful process and the learner has a number of possibilities to switch from one thinking level to another. Starting from level L1(a), the learner can be brought to level L1.1(a) through TS P1(a) and through lateral thinking, transition to level L1.2(a) takes place rather than through PS P1.1(a) which is at a higher level of thinking. Then transition to level 1.1(b) occurs. In the stage of transition, proper development of core constructs will assist the learner in moving from low level of thinking to higher order level of thinking, hence providing more avenues towards critical and lateral thinking. During the *transition stage*, core constructs are developed, but at different degree depending on the amount of challenges the learner is experiencing.

This route is a complex one as it involves critical and lateral thinking; the learner has the possibility to confront his/her initial sate of mindset. The transfer may be bi-directional depending on the existing state of thinking of the learner.

The development of core constructs between physics and mathematics ($\Leftrightarrow$) can be represented as follows:

L1(a)↔L1 (b) ⇔ L2 (a) - {TS P1 (b), TS M1 (a), TS M1 (b)}-↔L2(b)

This sequence is of low order but the link between mathematics and physics should be made during the teaching/learning process for proper understanding of physics. A more complex situation can be represented as follows:

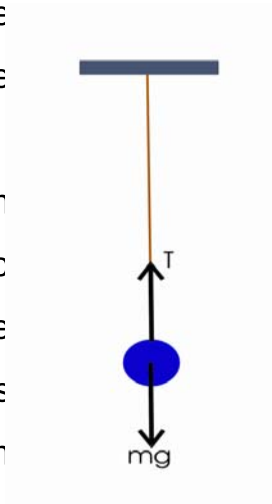
The learner needs to demonstrate ability to solve a contextual physics problem that require a number of thinking processes, for e.g. level L1.2.1(b) has to be reached through the following thinking processes:

L1 (a) ↔ L1 (b) – {TS P1 (b)} - L1.1 (b) ↔ L1.2 (b) ⇔ {TS P1.2 (b), TS P2.1, TS P1.2 (b)} – L1.2.1 (b).

This Thinking Process Model has formed the basis of our research activities as it has enabled us to determine the level of thinking of learners after prior knowledge has been tested and during the conduction of the physics lesson by one of the member of the research team, assisted by the other team members.

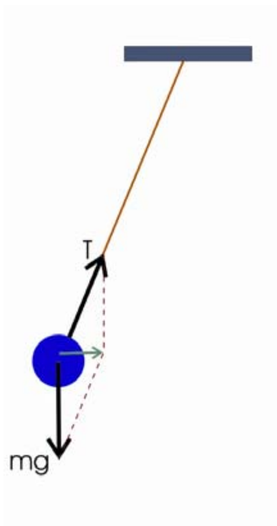
An example is henceforth provided:

1. The learners are brought to level state L1(a) when they are asked to draw the forces acting on the ball.
2. They are brought to level L1 (b) when they are asked to explain where the two forces should act and why. It is to be noted that it is necessary to lead students to concentrate on the ball rather than on the thread at the support. There will be an action-reaction pair at the point of contact of the thread with the support.



These two levels do not require much effort as the learners have already covered Newton's third law in Form IV-V.

1. The learners are asked to draw the forces when the ball is displaced to the vertical position. This is another thinking level, L1.1 (a). Level L1.2(a) can be triggered if learners consider the forces that act at the support.
2. The learners should now explain why the ball oscillates when released. The learners should be guided to develop core constructs, that is, they should be now in levels L2 (a) or L2.1 (a).



They should be able to link acquired mathematics knowledge with physics, that is, they should be able to relate the concept of 'resultant' with the two forces and also determine the direction (sign) of motion. This is represented by TS P2.1(a) and TS P1.2(b) which refers to the development of core constructs in physics and mathematics. Once the learners' thinking is geared towards *transition stages* TS P2.1 (a) and TS P1.2 (b), they are able to switch between the mathematics and physics.

As the level of difficulty in the question increases, the learners should show their abilities in developing core constructs and versatile ability to switch from physics to mathematics knowledge and vice versa. The teaching process should prone such a practice and it cannot be assumed that, in the first instance, students will develop competencies in transferring acquired knowledge from one subject to another. Once students acquire these skills, they will then be able to operate as independent learners.

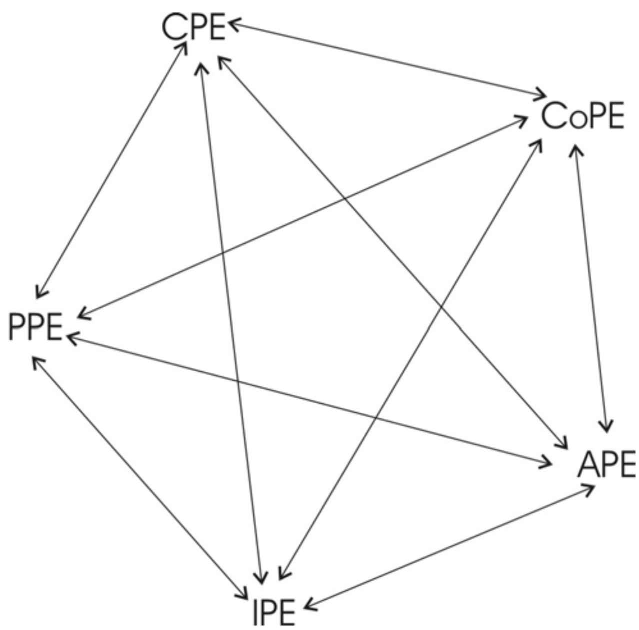
If the learners are sufficiently guided during physics lessons to develop core constructs and to make transitions from one thinking level to another, they will have opportunities to become independent learners and thus develop core constructs on their own.

The Comprehensive Interactive Process Model

The Comprehensive Interactive Model (CIP) of Evaluation has been developed by Parmessur et al. (2003) with a view to better evaluate understanding of students during the learning process. Six stages are involved in this model, namely,

- Conceptual process evaluation (CPE),
- Procedural process evaluation (PPE),
- Analytical process evaluation (APE),
- Experiential process evaluation (EPE),
- Conclusive process evaluation (CoPE),
- Summary process evaluation (SPE).

For the purpose of the research study only five stages have been used. Since this study was a pilot one, we encountered time constraints and therefore it was necessary to adapt the evaluation process according to the circumstances prevailing in the school.



The course of the lessons proceeded in such a way that students' understandings were constantly being challenged and they were guided to revisit their pre-existing knowledge and to make necessary corrections. Students were given the opportunity to constantly interact with the tutor as well as with themselves and this approach enabled us to determine the level of their reasoning power. Both the thinking process model and the comprehensive interactive process model have been instrumental towards achieving the aim of our study.

Chapter 6

Using core constructs for conceptual development

Lesson 1: School C2 (Girls)

Duration: 80 minutes

Concept taught: Kinetic Theory of gasses

Learning Outcomes

At the end of the lesson, students should be able to:

(i) demonstrate understanding of relationship between P, V and T,

(ii) relate amount of substance to the mole,

(iii) explain physical phenomena using the ideal gas,

(iv) deduce a formula for the pressure exerted in a gas,

(v) use the relationship between kinetic energy of molecules and thermodynamic temperature to explain physical phenomena.

Equation of Ideal Gas

After an introduction based on prior knowledge of students, Boyle's law, Charles's Law and pressure law were introduced.

To note that at all time, student's mathematical knowledge were invoked and in cases of misconception, additional questions were set to the student. Students were briefed that they can, at any time, ask questions for clarifications or if they have not understood.

1. Boyle's law

The volume of a fixed mass of gas at constant temperature is inversely proportional to its pressure.

(i) Students were asked to write down the mathematical formulation of that statement in terms of proportionality.

(ii) Students were then asked to re-write the same statement without the proportionality sign.

(iii) Students were asked to explain the meaning of the constant.

(iv) Students were asked to explain what happens when mass changes. Will that relationship still hold?

(v) Students were asked to draw the relationship between pressure and volume.

(vi) Students were asked to draw another similar relationship but this time for a different mass.

[Some students were experiencing difficulties to draw the graphs and the researcher conducting the teaching was at all times setting prompts (direct links with mathematics) so that students are led to the correct answers.]

Charle's law

The volume of a fixed mass of a gas at constant pressure is directly proportional to its thermodynamic temperature.

The same types of questions were set but this time students were asked to comment on this constant.

Pressure law

The pressure exerted by a fixed mass of gas at constant volume is directly proportional to its thermodynamic temperature.

Again similar questions were set and students were asked to justify the constant.

Students were then asked to combine the three equations into a single one. Most of the students got the correct answer since they have already come across the equation in Form IV-V.

Students were then asked to use a mathematical approach to obtain the same equation. Students were asked to clearly show the procedures. There is clearly a mismatch between mathematics and physics knowledge. *No one could work it out*.

The following prompts were set:

- *To write the equations with different constants;* $PV = k_1$*;* $\frac{V}{T} = k_2$*;* $\frac{P}{T} = k_3$*,*
- *To multiply LHS with LHS and RHS with RHS,*
- *To simplify the equation,*
- *To consider a different constant and to explain why this is possible.*
- *The last stage of the discussion was to enable students to come up with the equation of state,* $PV = nRT$ *.*

Students were then able to comprehend as this was visible by their reaction.

The above questions that were set were based on the comprehensive interactive process model that Parmessur et al. (2003) have developed (refer to chapter 5 and Appendix 5).

A formative assessment was set to probe further whether students still holds misconception.

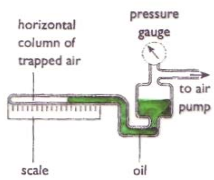
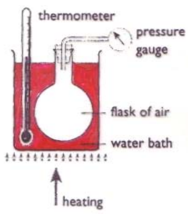
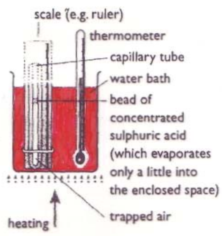
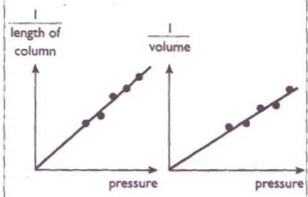
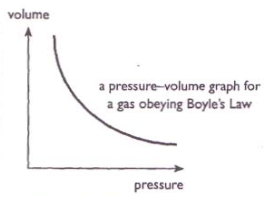
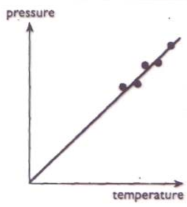
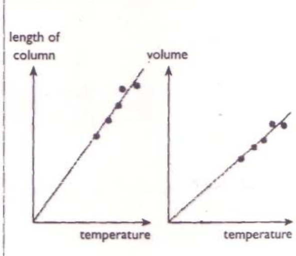
In the lung, the respiratory membrane separates tiny sacs of air (absolute pressure = 1.00 x 10^5 Pa) from the blood in the capillaries. These sacs are called alveoli, and it is from them that O_2 enters the blood. The average radius of the alveoli is 0.125 mm, and the air inside contains 14% O_2, which is somewhat a smaller amount than fresh air. Consider the body temperature to be 37^o C.

(i) Sate two assumptions that you need to consider in order to tackle the problem,

(ii) Calculate the number of O_2 in one of the sac.

It was very pleasing to note that students were able to solve the problem after some prompts were provided.

A short summary was provided to students as provided below:

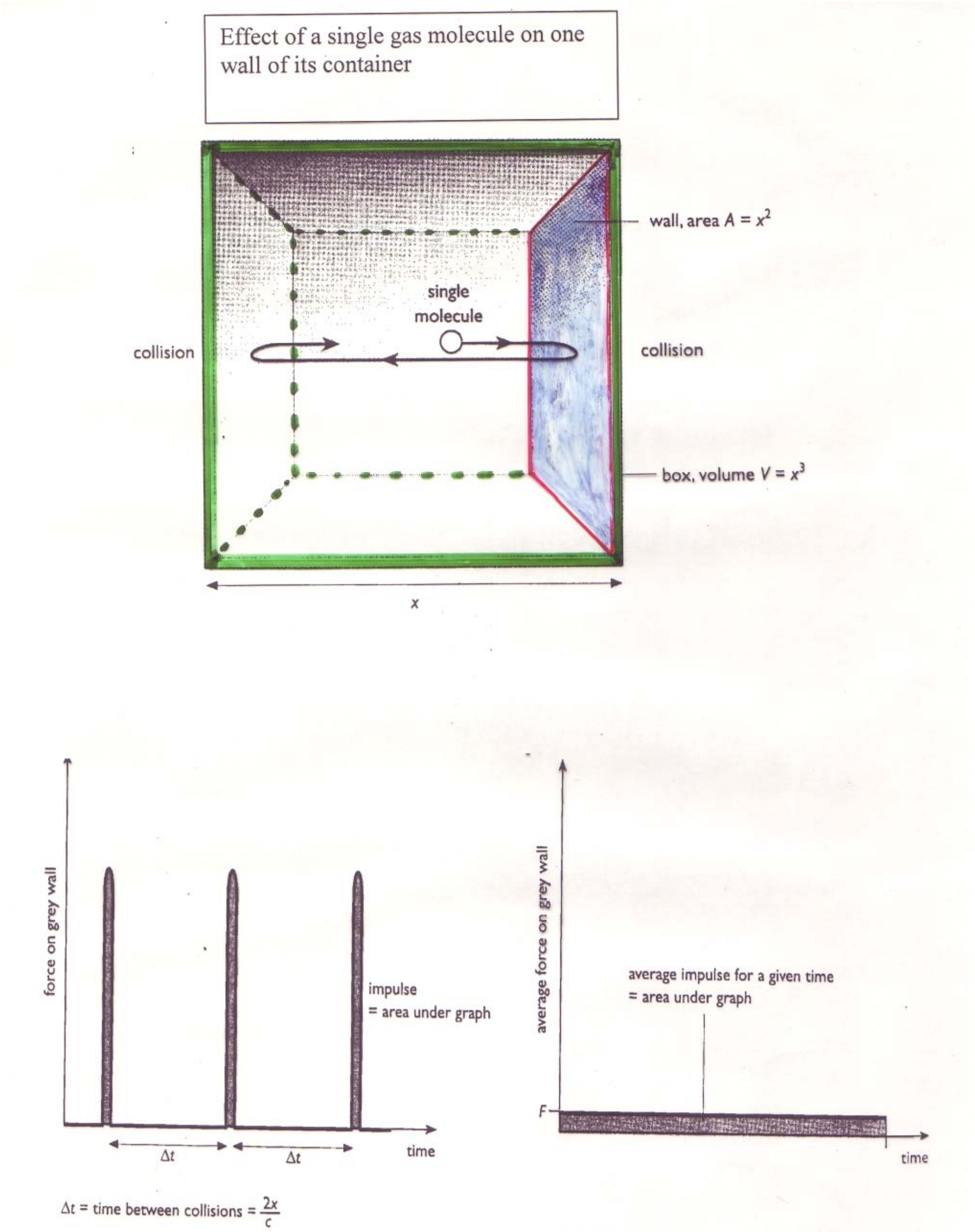
Gas Law	Boyle's Law	Pressure Law	Charles' Law
Input (independent) variable	P	T	T
Output (dependent) variable	V	P	V
Kept constant	m, T	m, V	m, P
Principles of measurement			
Graphical and mathematical representation	 $\frac{1}{V} \propto P$ $PV =$ constant 	 $P \propto T$ $\frac{P}{T} =$ constant	 $V \propto T$ $\frac{V}{T} =$ constant

The following assumptions that are required to derive the equation of pressure of a gas were provided to students and were asked to justify each assumption. Prompts were set to guide the students.

Assumptions

- The force of attraction between molecules is negligible.
  Students were asked to explain what would happen if this assumption was not taken into consideration. How this would influence the speed of the molecules if there are attraction.
- The volume of the molecules is negligible compared with the volume occupied by the gas.
  Students were asked to explain what would happen if this was not the case?
- The molecules are like perfectly elastic spheres.
  To explain what is meant by elastic spheres.
- The duration of a collision is negligible compared with the time between collisions.
  To explain what would happen if the duration was not negligible. How this would have affected the collision?

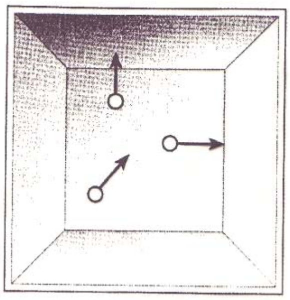
The next part of the lesson constituted in enabling the students to derive the equation for pressure exerted by a gas, $P = \frac{1}{3}\frac{Nm}{V} < c^2 >$ or $P = \frac{1}{3}\rho < c^2 >$. Students' knowledge of Newton's law of motion was challenged. Moreover, to come up with the above equation students needed to demonstrate mastery of mathematical ability to perform operation in geometry and trigonometry. There is a need to point out the questioning techniques adopted by the tutor enabled all the students to participate; even wrong answers were welcomed. In this way, such a strategy enabled even the shy learners to interact (Selley, 2000). Harper et al. (2003) consider that a weekly report from students will enable the teacher gain insight in students' learning process. Students will thus be encouraged to develop skills in question-asking since *active learning* is essential for understanding.

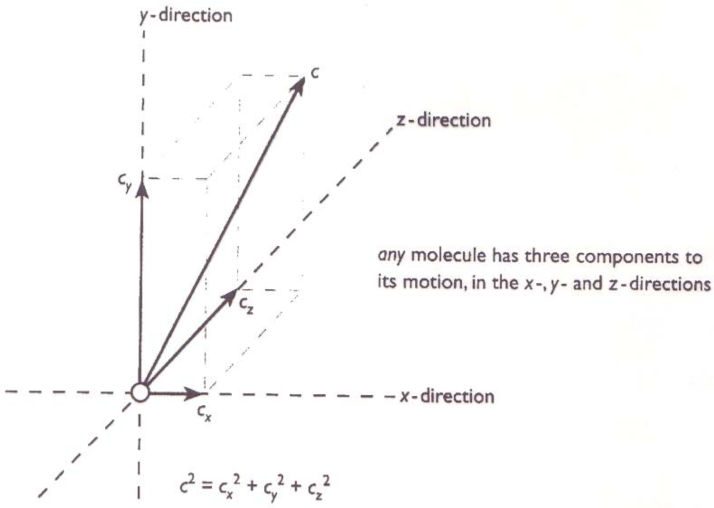


This slide enabled students to confirm the fourth assumption. They were asked to interpret the graph and to relate all the data that can be gathered from the two graphs.

Here, students' mastery of components of vectors was fully investigated.

For each molecule, velocity has three components. For many molecules moving in all directions, the maths is the same as for a set of particles of which one-third are moving in the *x*- direction, one-third in the y - direction, and one-third in the z - direction.





Students' knowledge in two dimensions was extended to three dimensions. They were expected to show how Pythagoras theorem is applied to two situations involving two coordinates and then extended to three coordinates.

A conclusion was provided and students' were encouraged to ask questions for further clarifications.

As summative evaluation, a few questions were set for homework and students were requested to critically analyse the lesson and to attempt the questions.

Lesson 2: School C3 (Boys)

Duration: 80 Minutes

Concept Taught: Electricity – Kirchhoff's Laws

Learning Outcomes

At the end of the lesson, students should be able to:

(i) Identify sign and symbols used in electric circuits,

(ii) Demonstrate understanding of current and potential difference,

(iii) Explain why a current flows in a circuit

(iv) Apply Kirchhoffs' laws in problem solving,

Before starting the lesson as per the learning outcomes, students' prior knowledge was probed into to find out any misconception. This enabled the researcher to alter his strategies during the course of the lesson development. Students' knowledge about sign and symbols were tested through questions.

The next phase consisted in enabling students develop core constructs in current flow in a closed circuit. In order to draw an analogy with current flow, students were briefed about water flow in connecting pipes and a series of questions were henceforth set:

When does water flow in a pipe?

If there was no difference in height, will the water still flow? [Comparison with an object falling in a gravitation field was invoked]

What is current? [Reference to Form V knowledge was made]

What constitute a flow of current?

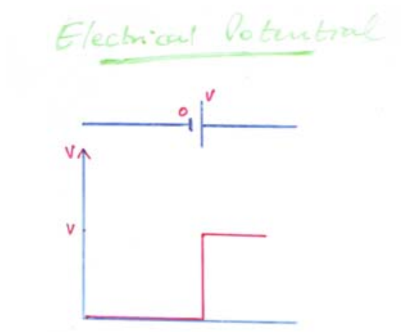
[it is important to enable students to reject the notion that the current leaving a bulb, for example, has less energy than before entering the bulb]

What is meant by potential at a point? [Reference to Form V knowledge was made]

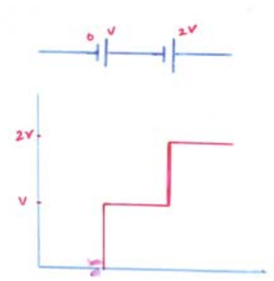
What is meant by electric potential?

All the above questions form part of the strategy that the researcher was adopting to provoke critical thinking that will enable the students to develop core constructs. Adequate prompts were provided to lead the students towards the formulation of correct answers. The students were themselves responsible for the learning of concepts.

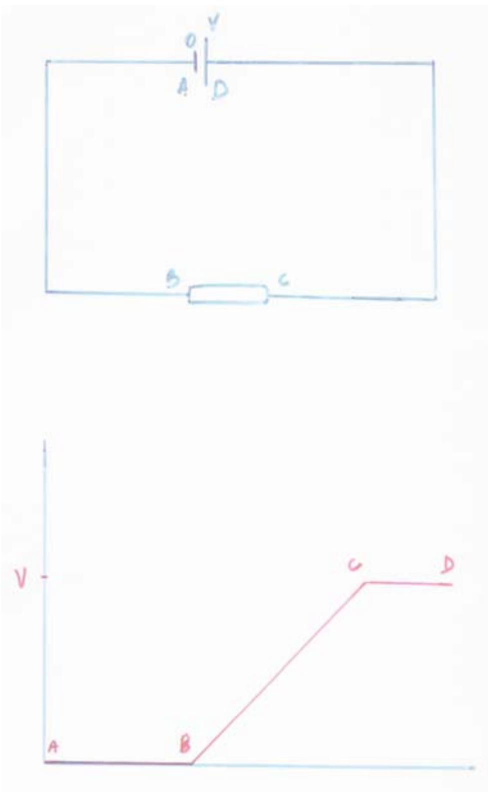
The next phase consisted of identifying misconception in interpretation of graphs and bridging the gap between mathematics and physics.



This slide illustrates the strategy that was adopted by the researcher in enabling the students to link mathematics and physics. Through probing questions it was shown how the potential varies from the negative terminal to the positive terminal of the cell.



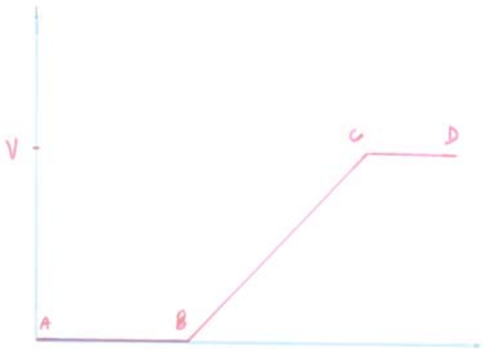
The students were then asked to draw a similar relationship when two cells are connected in series. Most of them could attempt this question.



In this particular case, students were lead to determine how the potential in the circuit varies. Through probing questions, the students were able to determine the relationship. Students were also encouraged to suggest equations for various regions in the figure. This activity enabled them to link prior knowledge in mathematics with physics.

This slide comprises of a formative assessment on the same concept, but this time in a different context.

With some difficulty, most of the students were able to obtain the correct answer.



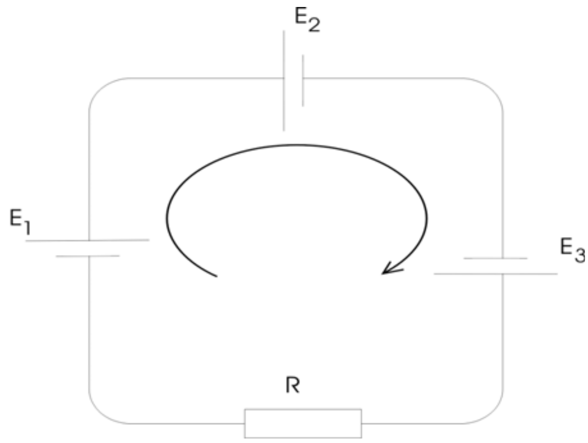
The following phase consisted in enabling students to develop a sound understanding of Kirchhoff's' laws. Students were provided with the opportunity to verify the laws and some challenging questions were set to reinforce acquired knowledge: *Is charge conserved? (Kirchhoff's first law); students were asked to provide evidence for their statement. Students were asked to explain and justify what will happen to the magnitude of the current if there is a resistor in its path. Students are led to conclude that* $\sum_{i=1}^{n} I_i = 0$.

These questions, in one way or the other enables the learners to revisit their conceptions of 'already acquired scientific knowledge'.

The next phase was to enable students to demonstrate understanding of Kirchhoff's 2nd law. The following questions were asked:

In which direction will the net current flow? Justify your answer.

There is a need to emphasise that students were allowed to challenge their peers answers and this strategy created a synergy in the classroom transactions.



This slide comprises of a formative assessment on the same concept, but this time in a different context.
With some difficulty, most of the students were able to obtain the correct answer.

Could the direction of the loop have been chosen in the opposite direction? Explain.
How is direction represented mathematically?
What is the sum of emf?
What is the sum of current?
Is it right that $\sum emf = \sum p.d$ *?*
The above equation is based on which law?

Students were asked to repeat the above process when the direction of the loop has been altered. Students were pleased to determine that they obtained the same answer. Two students required more attention probably because of language problem as was stated by their teacher. The lesson ended with a summary of the main points involving the mathematics and physics linkages and students were invited to ask question pertaining to the lesson. From the reactions of the students, it was clear that they liked the lesson. The written feedback obtained will be analysed.

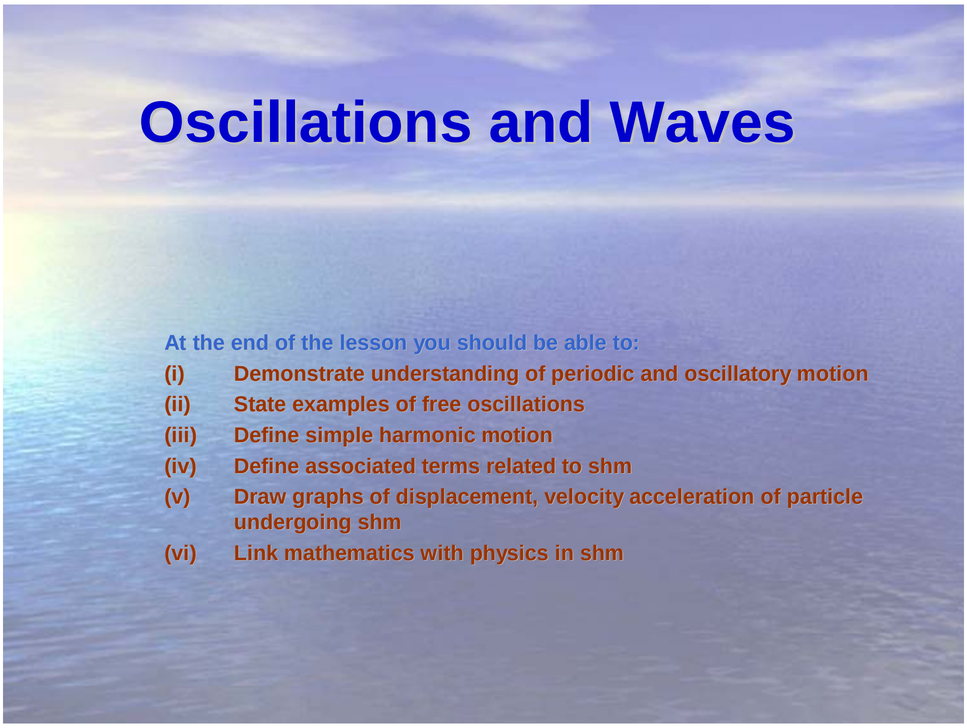
Lesson 3

The aim of this lesson was to enable students to develop a sound understanding of the concept of simple harmonic motion. Mathematics is a prerequisite to the understanding of the concept. The teaching was done using power point presentation so that students are to visualise some of the phenomena associated with simple harmonic motion.

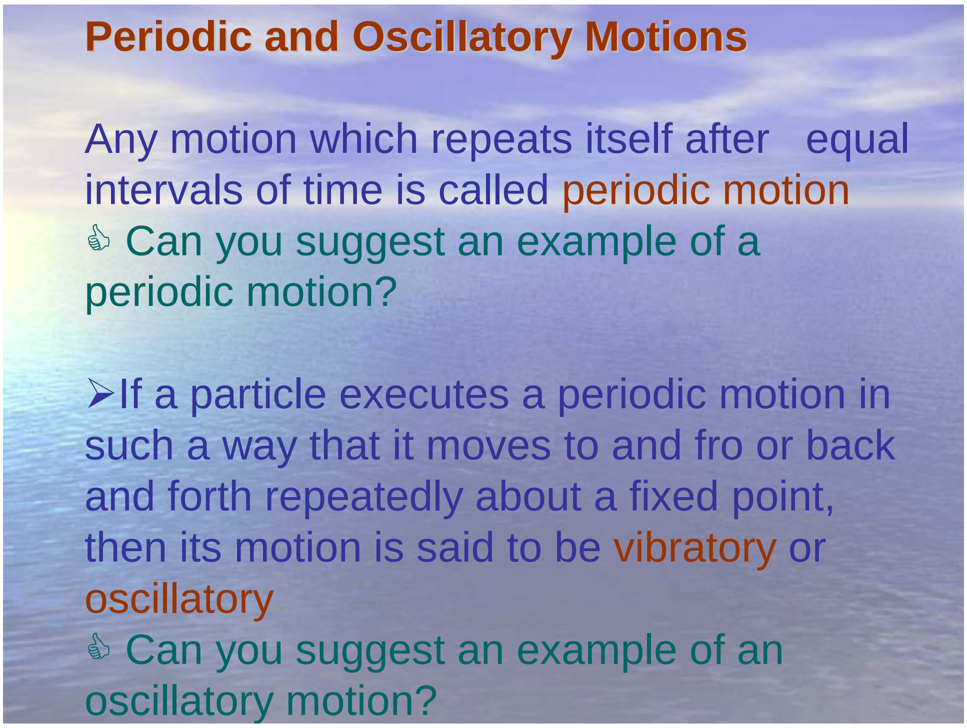
Lesson 3: School C3 (Boys), C4(Boys), C11 (Boys & Girls)

Duration: 120 minutes

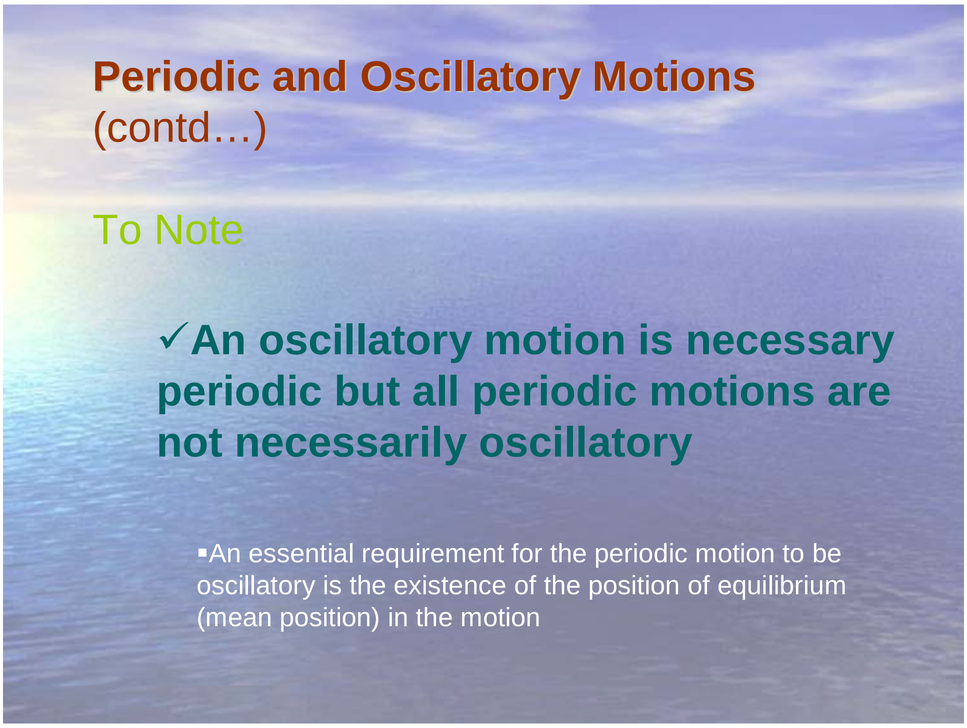
Concept taught: Simple Harmonic Motion

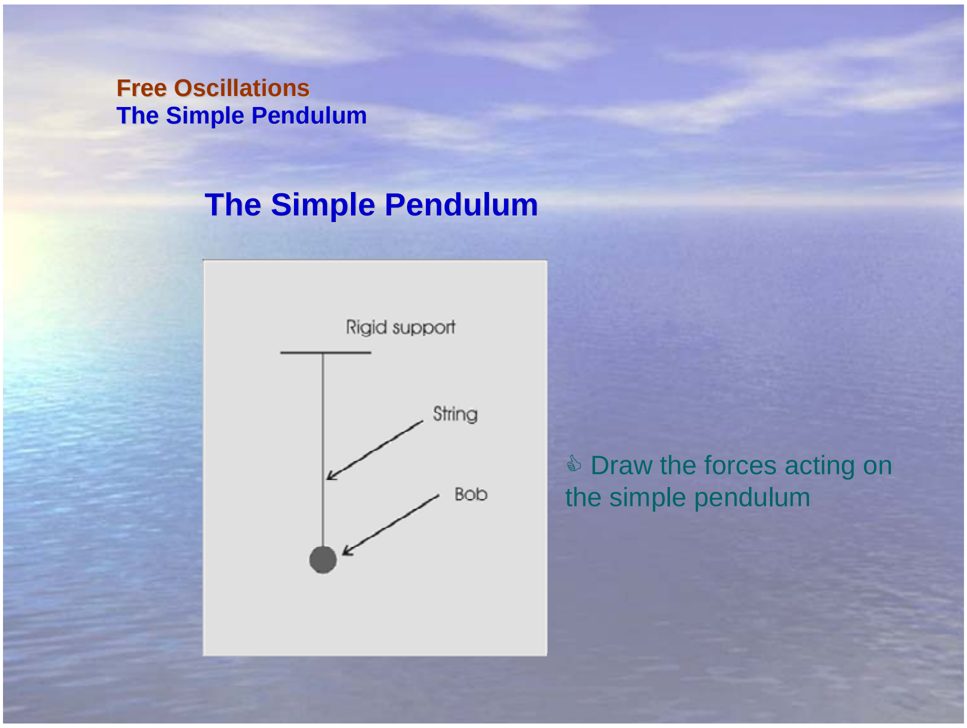


It is imperative that any lesson should start with informing the learners about the learning outcome, which intends to inform them about what they will learn during the present lesson.

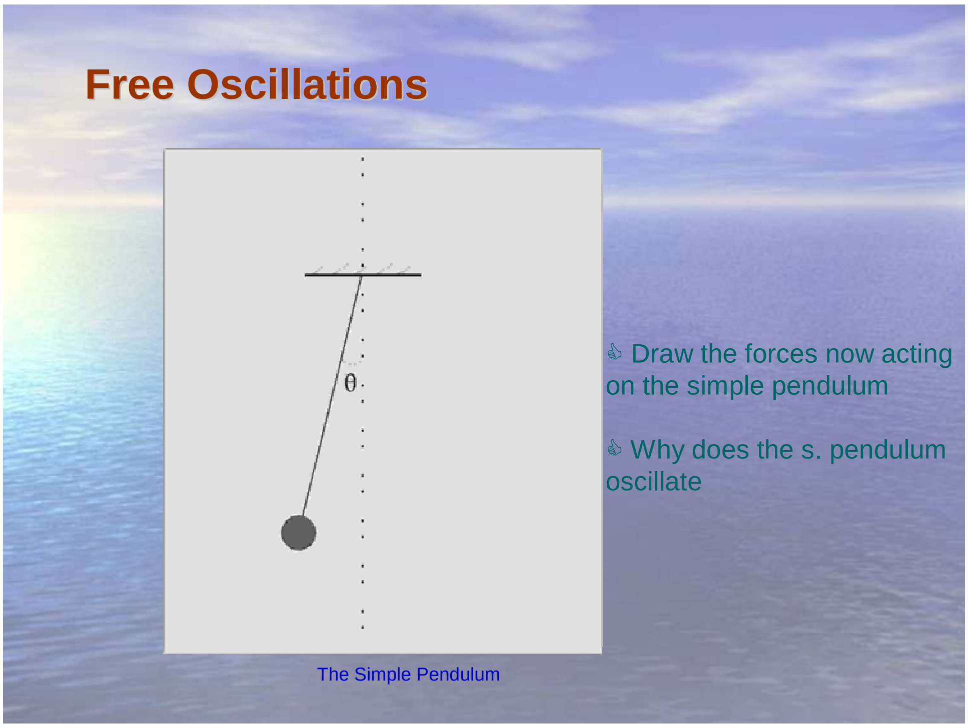


The questions that are set enabled the students to link prior knowledge with periodic and oscillatory motions and at the same time, the researcher could refocus his strategy. Learning is enhanced when opportunities are created for individual learners to develop conceptual understanding. Carin and Bass (2001) are of the opinion that teachers have to help learners to bridge links so that connections are made reversibly.





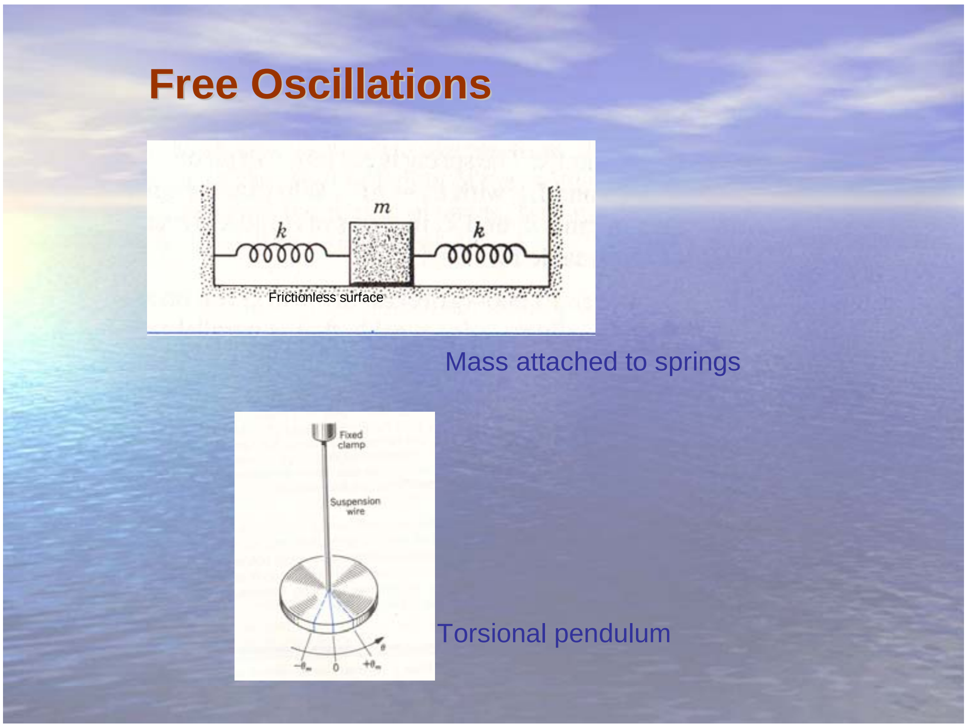
All students were able to draw the forces (weight and tension) acting on the simple pendulum.

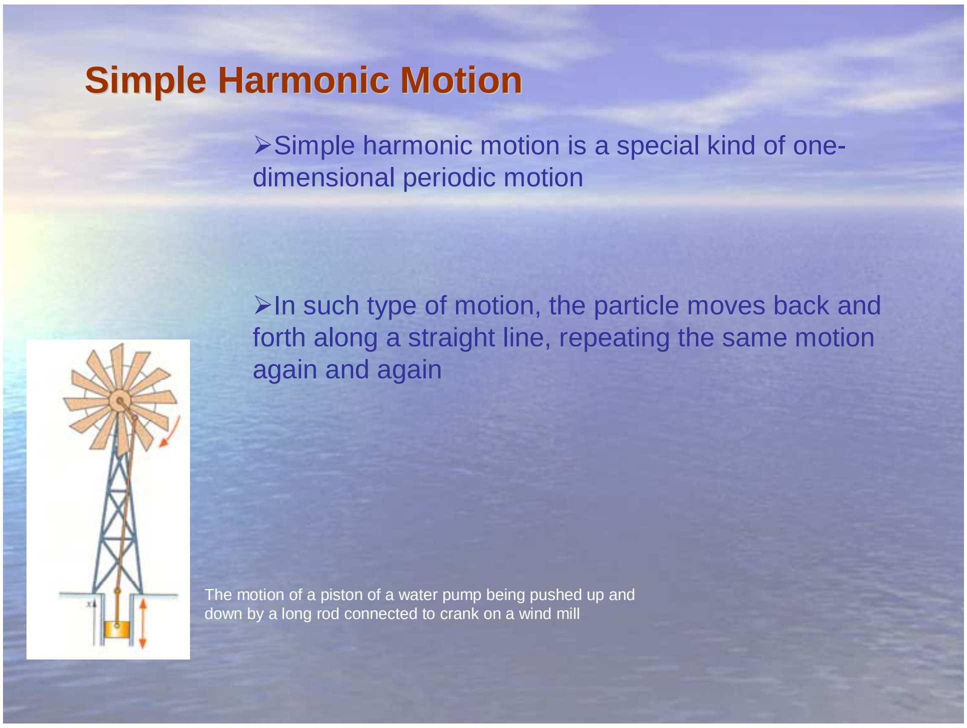


These two tasks enable us to determine students' understanding of balanced and unbalanced forces. Students' understanding of Newton's 2nd law was also probed.

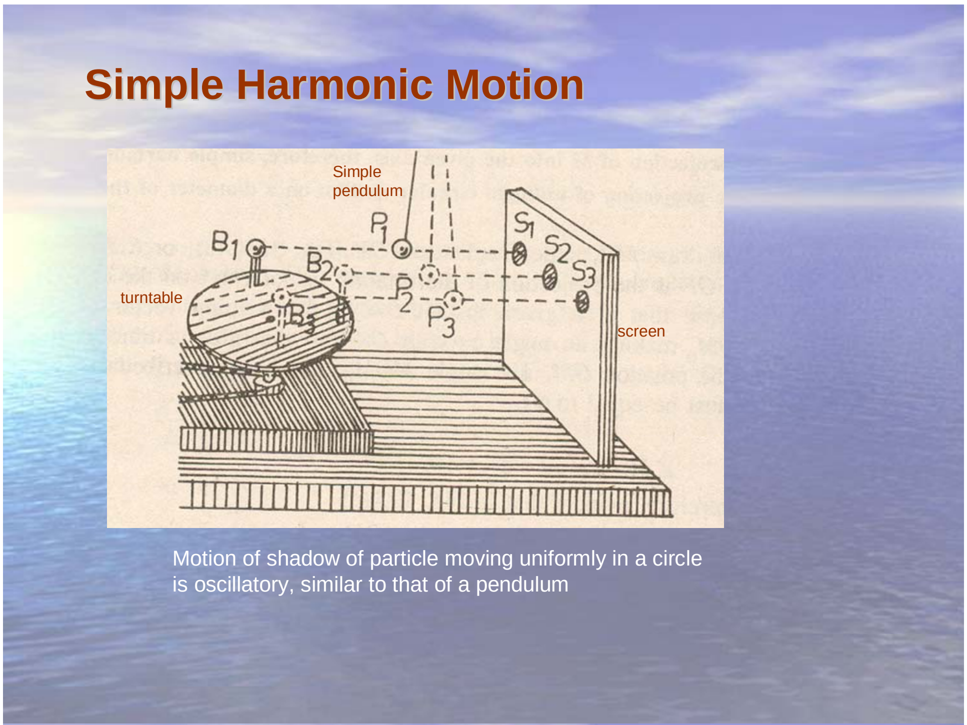
It is to be noted that this question posed a number of problems to the students. Though most of them could identify the forces, none of them were able to state what caused the oscillation to occur which is quite surprising as they have already learnt about laws of motion.

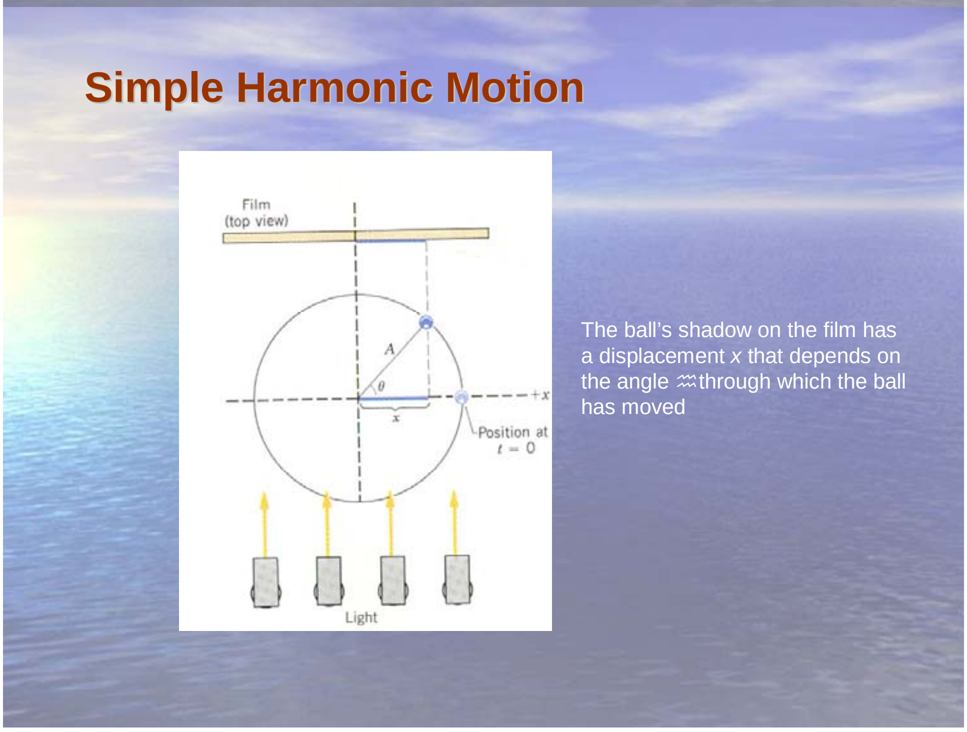
Students were lead to link their mathematical knowledge and to use the vector approach to explain the occurrence of a resultant force. Students were helped on an individual basis depending on their responses. The strategies adopted during the lesson development enabled us to capture the operative attention (Schon, 1987) of the students so they are able to act appropriately according to the instructions.



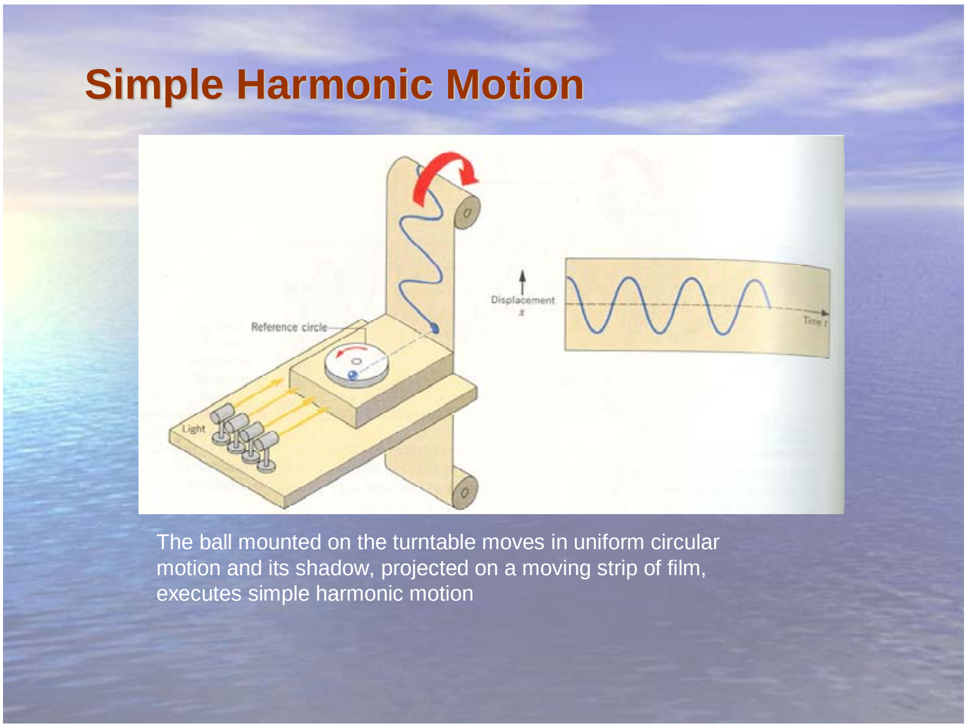


More examples on free oscillations were set for students to firmly grasp the concept.

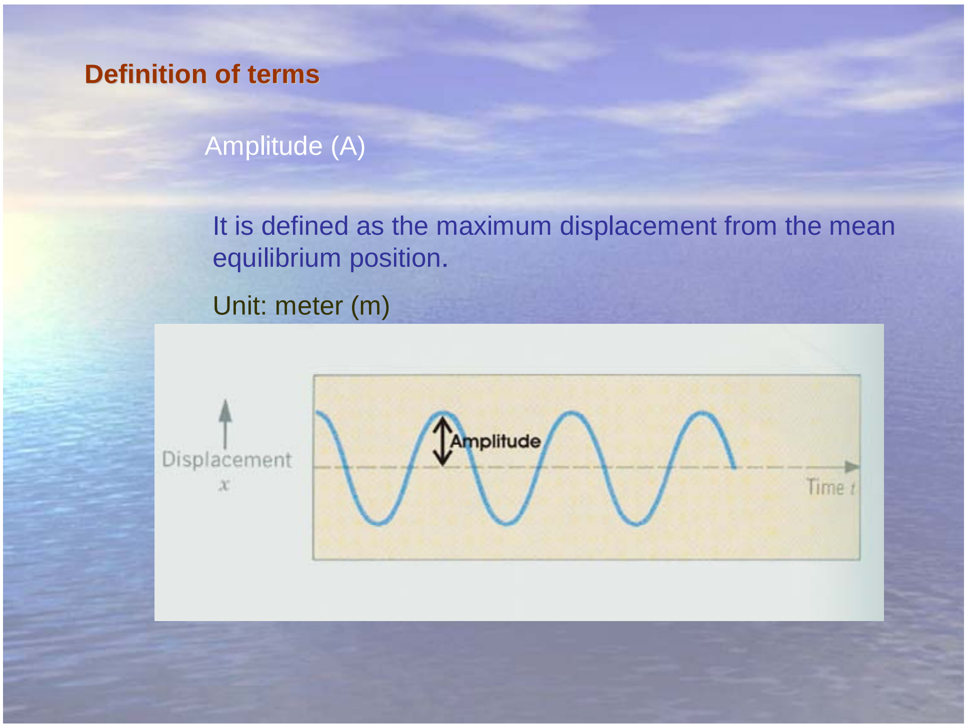




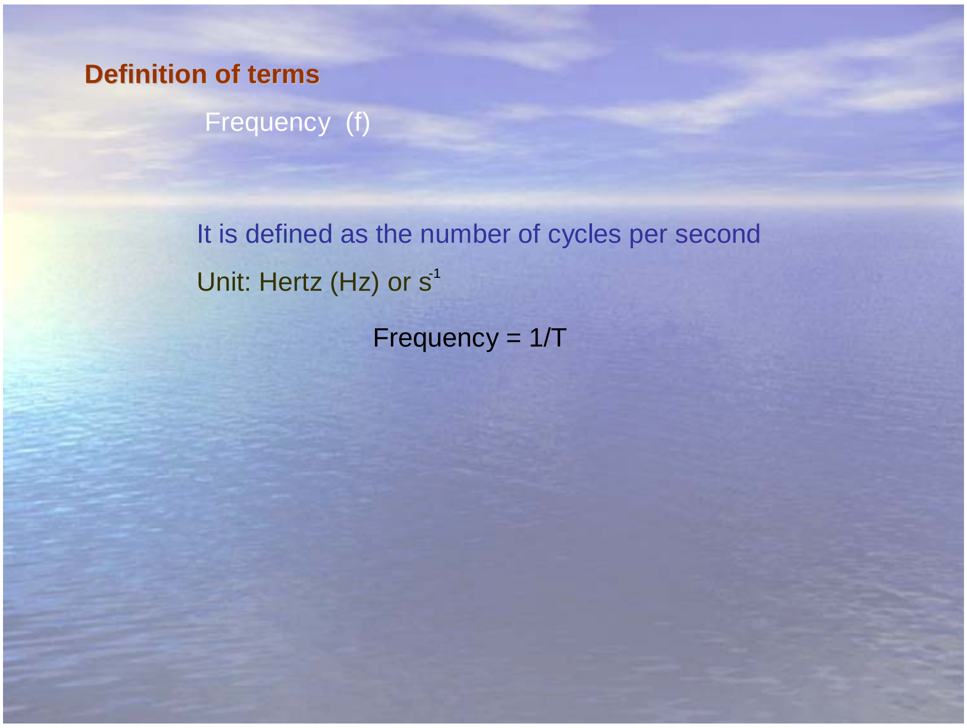
Students were asked to determine a mathematical relationship for the motion of the shadow in the horizontal plane. Their knowledge of trigonometry was important here. It is to be noted that at each stage of the lesson, students were led to the correct answer.

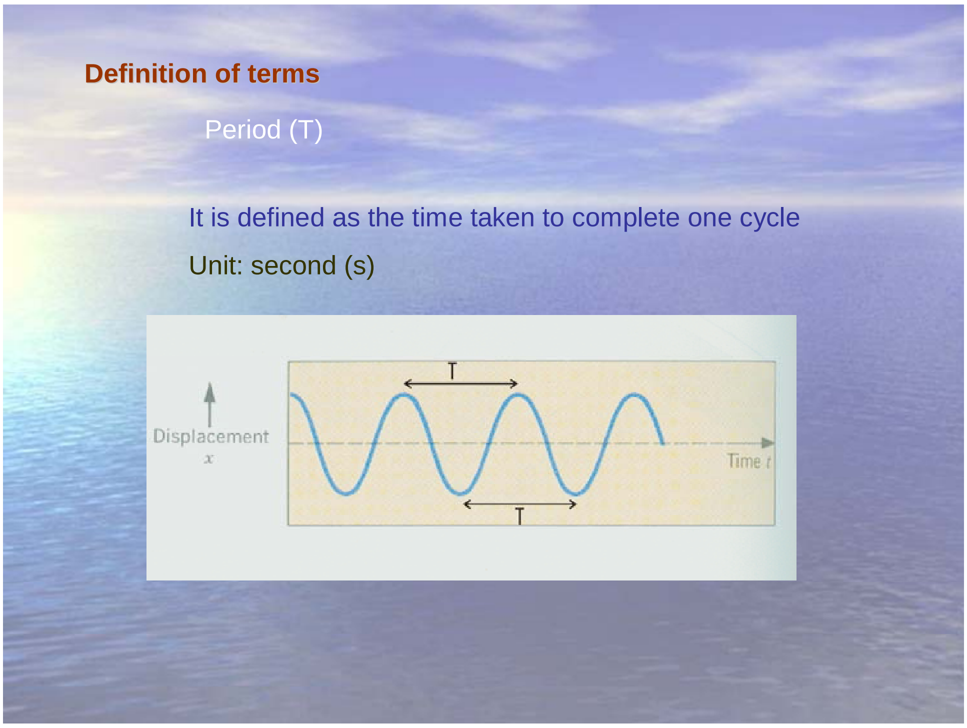


Students were shown how a sinusoidal graph could be obtained when a trace was made continuously on paper in a horizontal direction and the paper was slowly pulled in a vertical direction. The students were asked to perform this task in pairs and when asked whether they have ever determined such a graph, the answer was unanimously. There is a need to mention that at Form IV-V students have covered the topic on cathode ray oscilloscope and that they should have developed understanding on how motion of the electron beam in the x and y axes contribute towards the sine relationship.

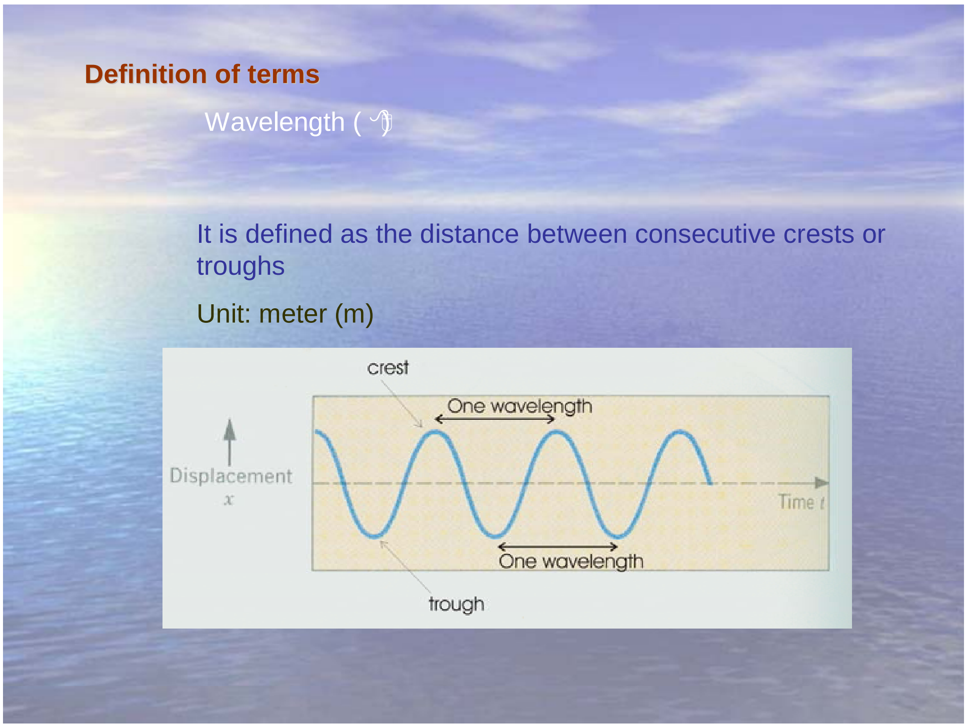


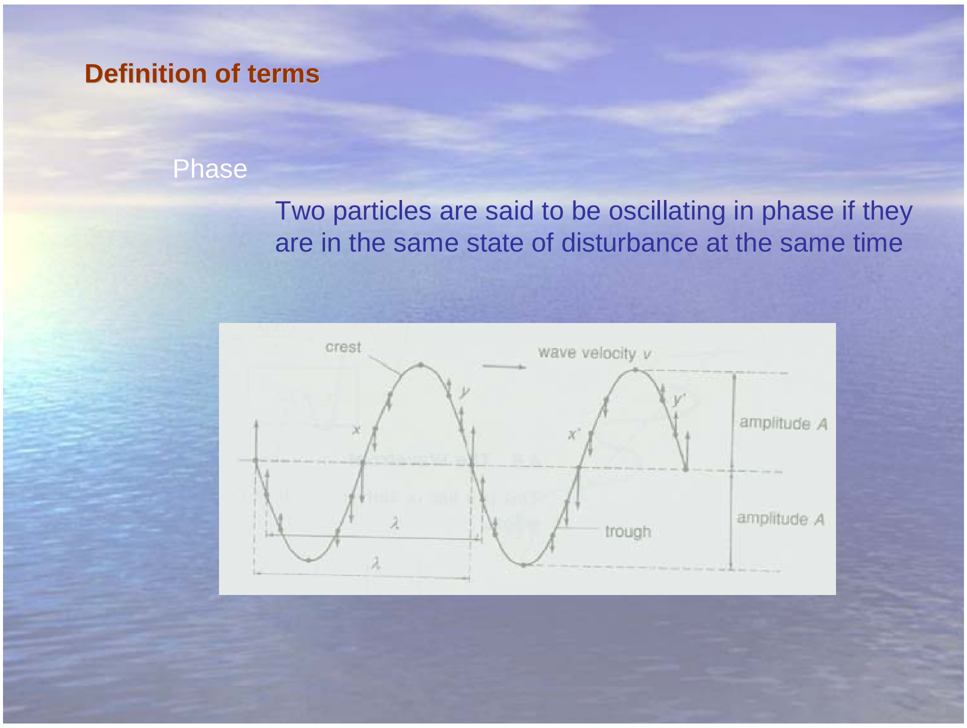
The slides which concerns recall of information of knowledge already developed in form IV-V were necessary to enable the researcher to unveil any misconception and thus enables them to him to alter his strategies as he proceeded. Misconceptions are categorized as *preconceived notions* (conceptions resulting from every day experiences), *nonscientific beliefs* (initiating from non scientific education – religious beliefs for examples), *conceptual misunderstanding* (faulty models are constructed with insecure base) and *factual misconceptions* (issues learnt *in childhood and retained in adulthood*).



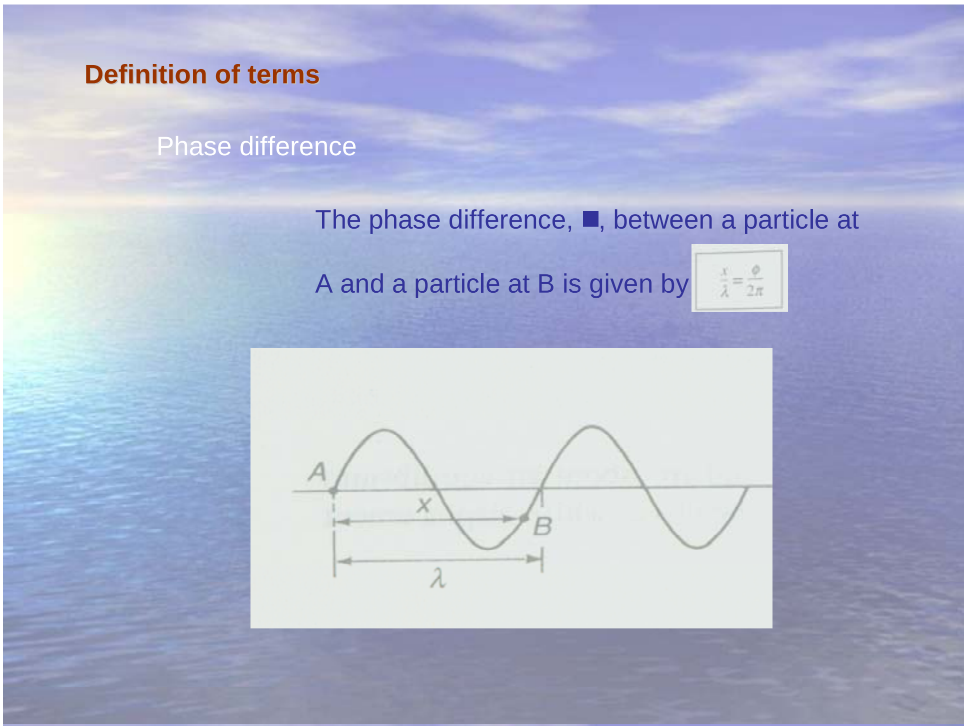


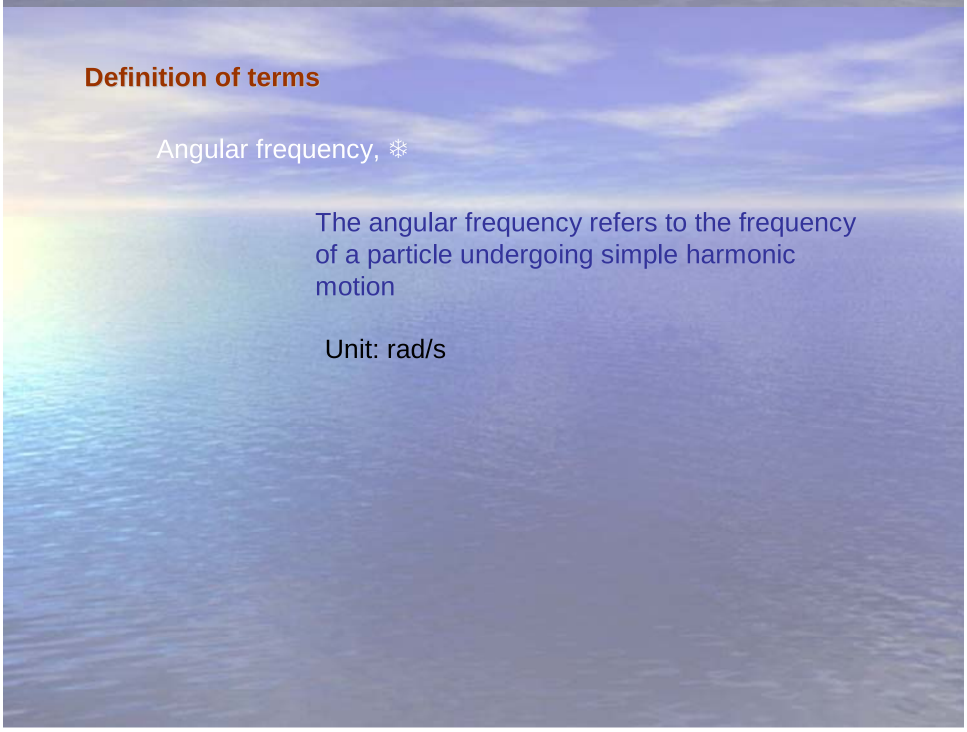
These two slides were meant to remind students of the concepts of frequency and period together with their respective units.



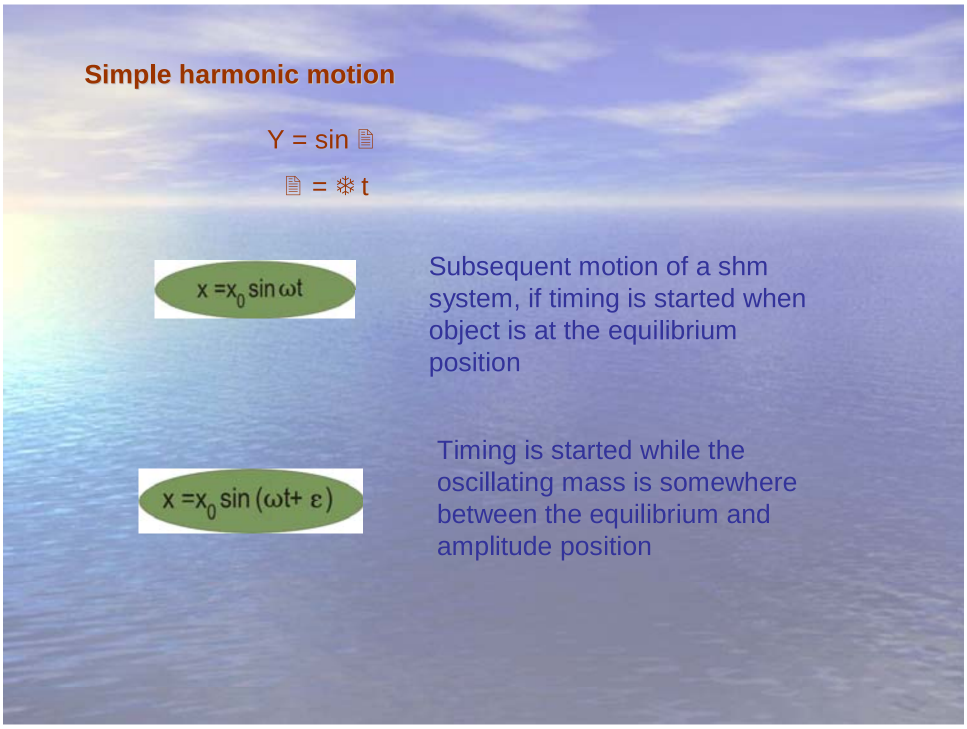


In mathematics, students have been solving a wide range of problems involving angles, but when it comes to solving problems in physics involving phase, they have difficulty. Students were led to come up with the notion that angle and phase are similar terminologies.

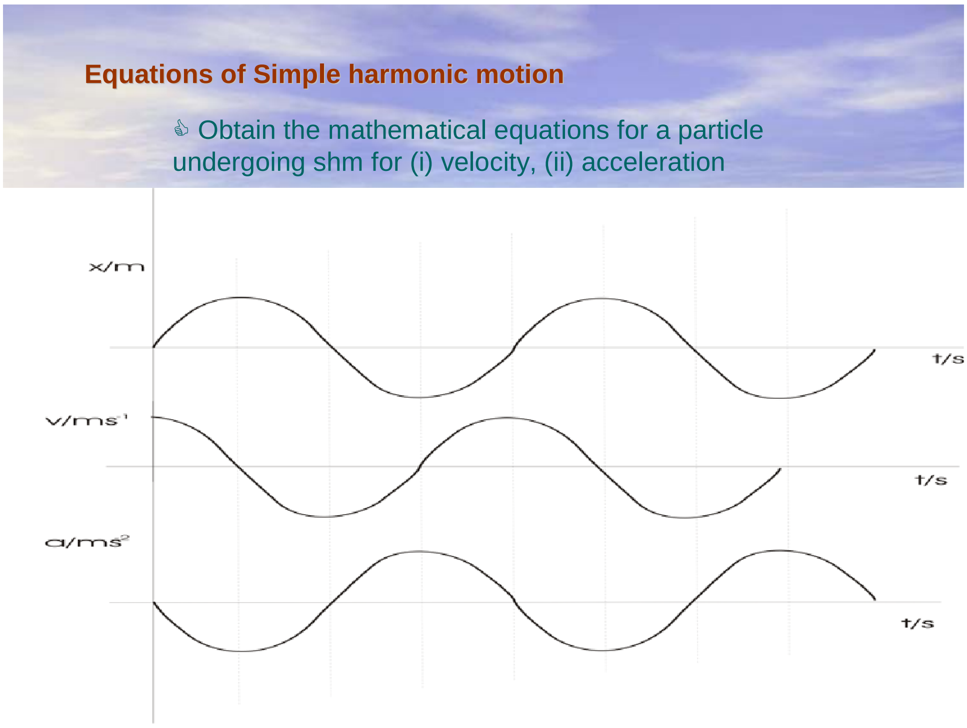




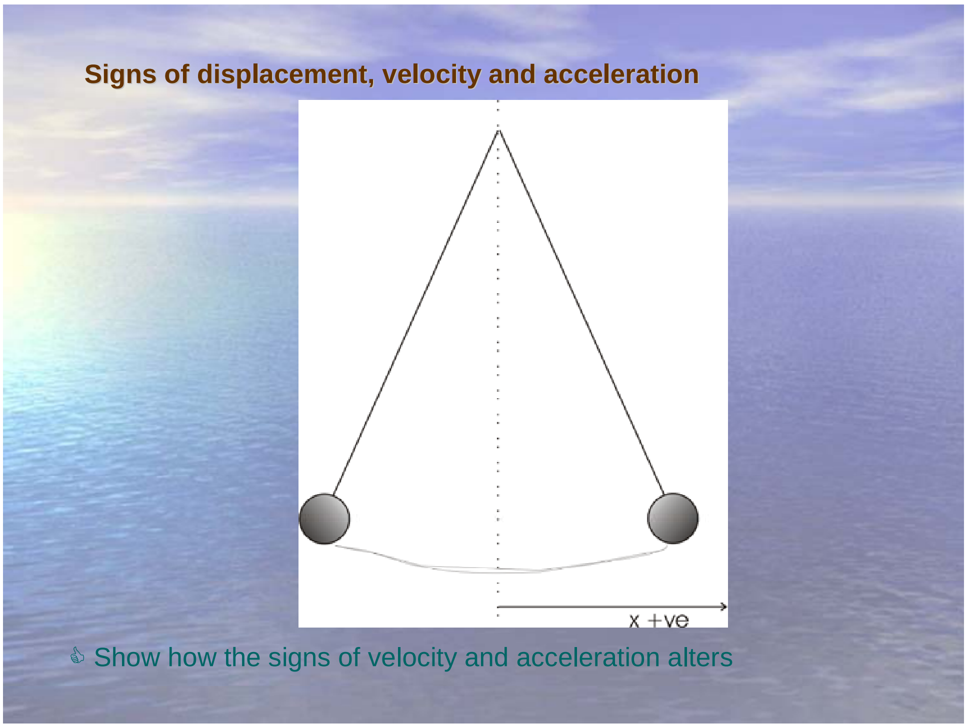
Students were asked to explain the difference, if any, between angular frequency and angular velocity which they had encountered in the topic of circular motion. It was a real dilemma for them to answer this question. To note that both concepts have the same unit of measurement.



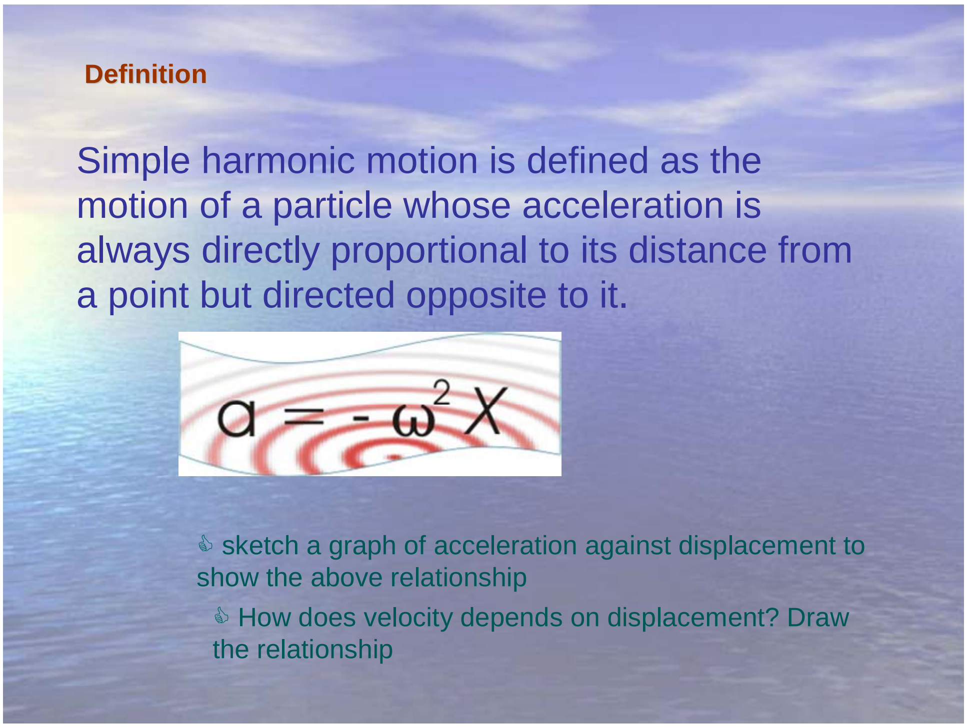
Students were involved into bridging the gap between physics and mathematics. Starting from the equation $y=asin\theta$, students were asked to explain why they will use henceforth the equation $x=x_0 sin\omega t$. They were asked to link their knowledge of circular motion to justify their answer. Hints were provided to focus students' attention on the time dependence which is a necessary variable in the equation for simple harmonic motion.



Students were encouraged to apply acquired knowledge of mathematics to write down the equations of the above curves. They were guided towards the answer for the first curve and they could easily work out the equations for the remaining two curves. It is to be noted that the low ability students were still experiencing difficulty and they were given individual attention until they reached the operative mode. This approach adopted by the researchers is quite demanding (Mortimer and Scott, 2003) and it enabled the students to develop core constructs for conceptual understanding.



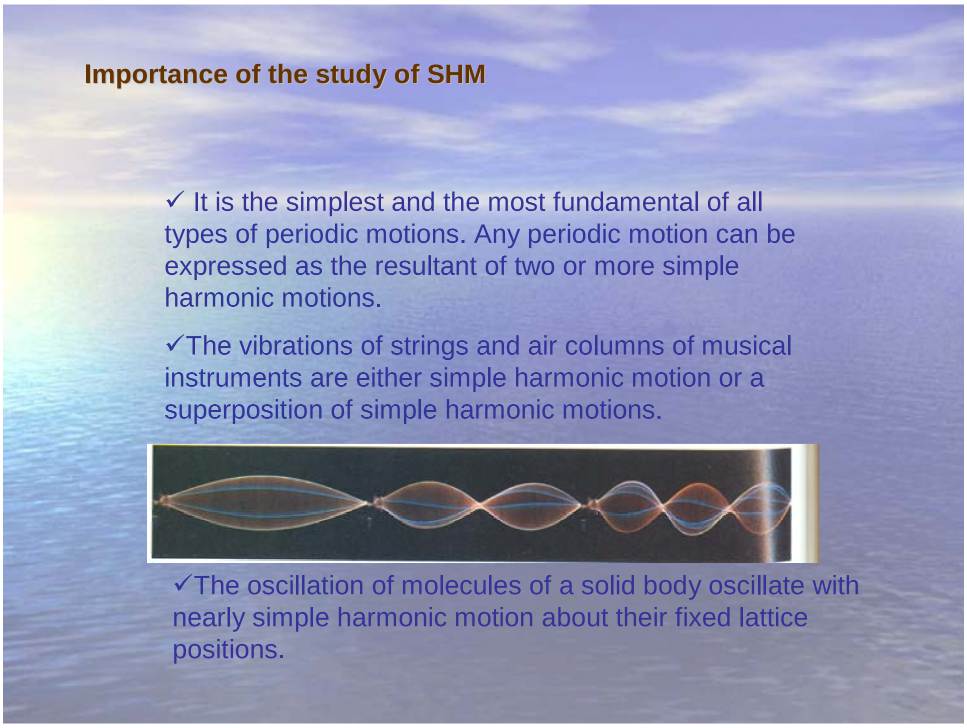
Interpretation of sign in physics is very important as it enables one to physically situate where the position of the bob is. Whenever a vector approach is applied, it is important for the learners to consider which direction is to be taken as positive or negative. It has to be noted that the selection of appropriate sign is a question of convenience that the students have to master to avoid confusion.

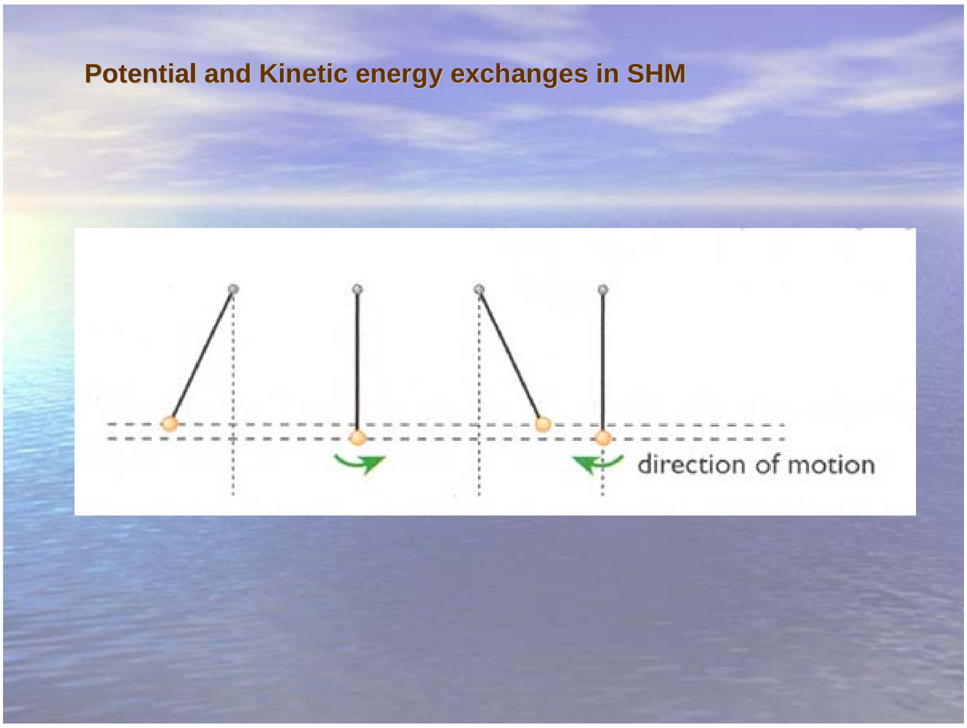


The above equation is linear provided ω is constant. Students were asked to justify the above statement. This equation is similar to $y = -mx$, a relationship which students have encountered since Form III.

Hints were provided to students so as to derive the equation for velocity:

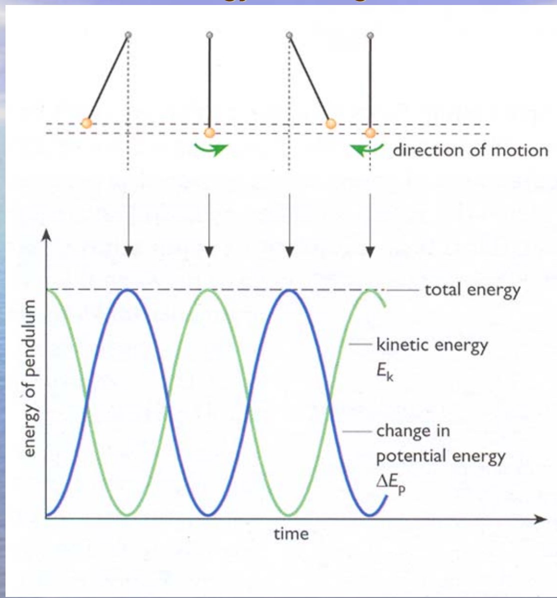
Starting from $x = x_0 sint$ and using differentiation and to recourse to identity $cos^2x + sin^2x = 1$, students were guided to obtain the equation for velocity ($v = \pm\omega\sqrt{(x_0^2 - x^2)}$) of a particle undergoing simple harmonic motion. This task was really hectic for the students as they were experiencing difficulty due to the presence of the square root sign. Hey were given step-by-step instructions to be able to draw the relationship between v and x.





Students' understanding of potential and kinetic energies of a ball falling under gravitational influences (without air resistance) is invoked to reveal any misconceptions.

Potential and Kinetic energy exchanges in SHM



In this case, though motion is in the positive as well as in the negative direction, the kinetic and potential energy curves are positive. An explanation based on the vector approach was sought from students.

Potential and Kinetic energy exchanges in SHM

E = K.E + P.E

K. E = $\frac{1}{2} mv^2$

P. E = $\frac{1}{2} m\omega^2 x^2$

Damped and Forced Oscillations

Due to frictional forces acting on the system, the amplitude of oscillation will not be maintained.

The oscillations will be damped. Such a gradually decreasing oscillation is called damped harmonic motion.

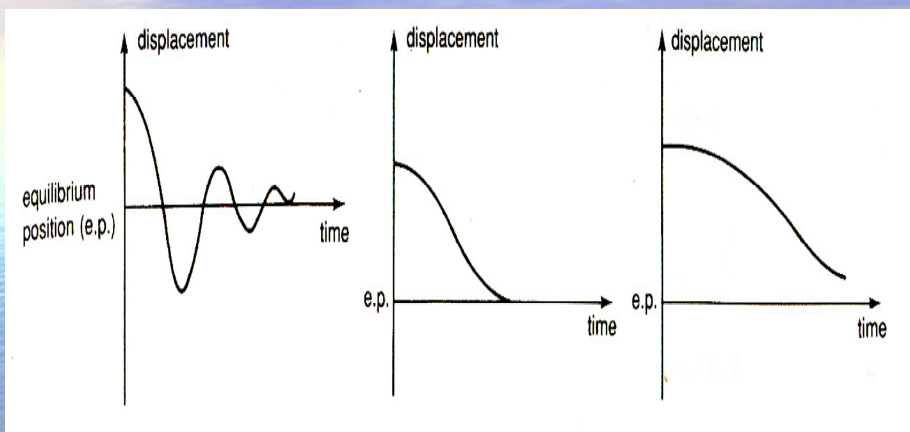
There are three types of damping:

✓ Light damping

✓Critical damping

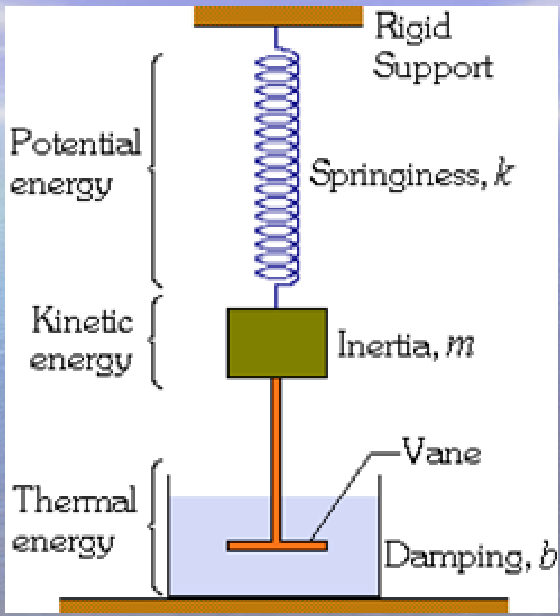
✓Heavy damping

Damped and Forced Oscillations



Students were expected to explain in full detail the behaviour of oscillating objects in relation to the above curves.

Damped and Forced Oscillations



Damped and Forced Oscillations

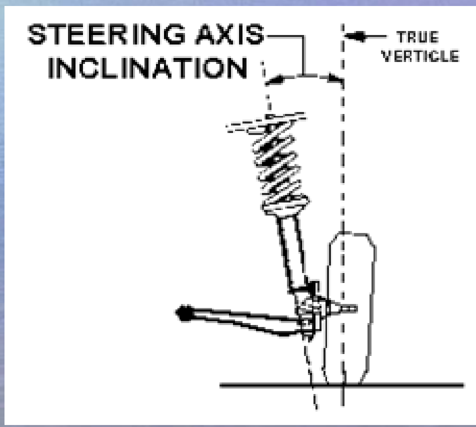
Importance of critical damping

✓ suspension system of a car

Damped and Forced Oscillations

Importance of critical damping

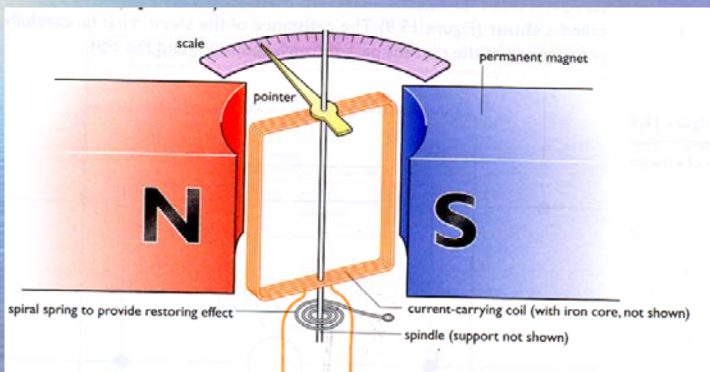
✓ suspension system of a car

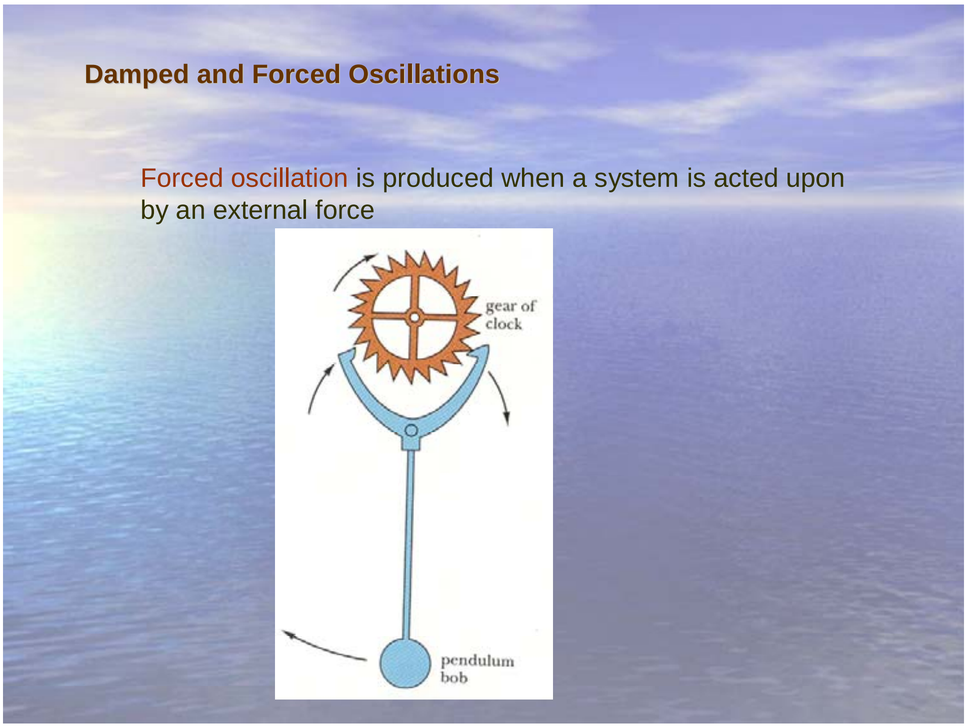


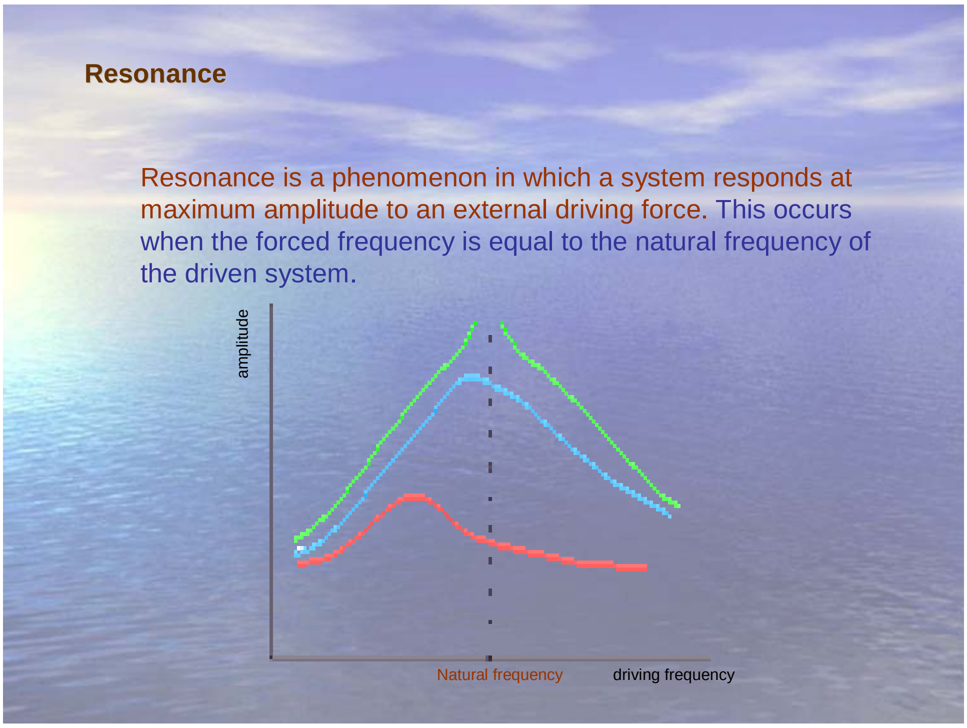
Damped and Forced Oscillations

Importance of critical damping

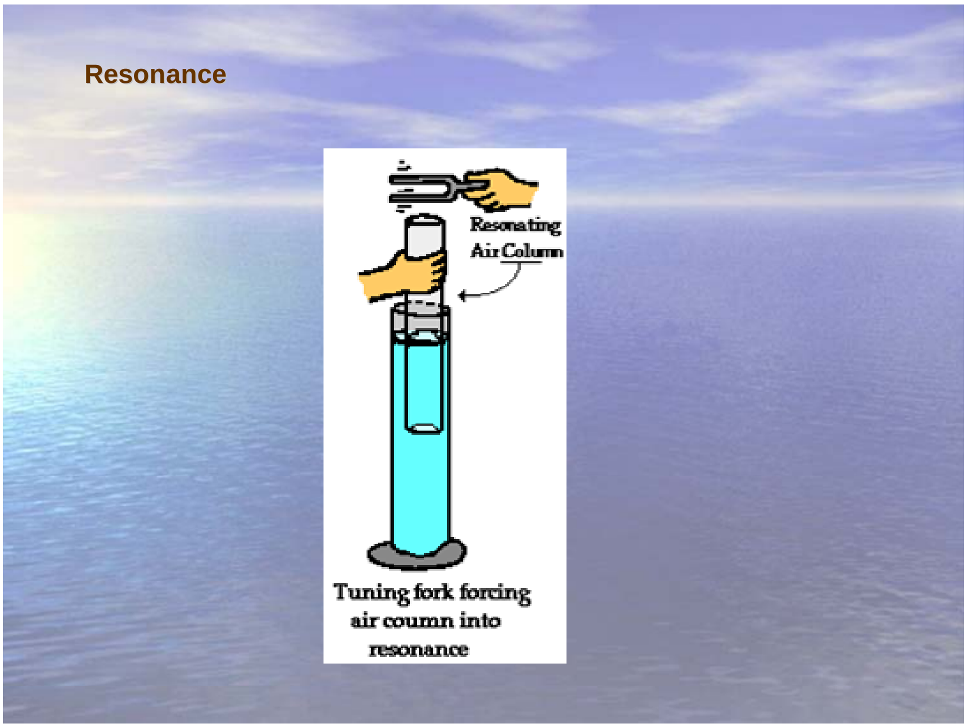
✓ ammeters, voltmeters







Here also students' knowledge and understanding of graphs were probed into.



This lesson on simple harmonic motion provided us with ample opportunities to test students' mastery of concepts in mathematics and physics. It is sad to say that most of the students did experience conceptual difficulty when it came to making linkages between mathematics with physics. As already mentioned earlier, it appears that students are not involved into developing strategies that would enable them construct meanings on their own, obviously, with adequate and appropriate guidance from the teacher.

After the teaching sessions, the researchers met and critically analyse their observations. The following consensus was noted:

- enthusiasm of the students
- interaction between the researcher – students, students - researcher and students – students
- prompts from the researcher led to re-oriented and focusing the animated discussions

- appropriate use of the Comprehensive Interactive Process Model assisted the researcher in quick identification and treatment of misconceptions
- adequate wait time were given to the students (for reflection and peer discussion)
- wrong answers were not discarded; they were used as a basis for further discussion for cross examination
- Opportunities provided to students helped them to link theoretical concepts to real life situations
- The objectives, clearly emphasized at the beginning of the lesson by the researcher, were helpful in enabling the students to summarise their own learning.

After the lessons, a questionnaire (see Appendix 3) was administered to all the students in the class to obtain their feedback on the strategies adopted in the lessons, to what extent they related mathematics to their learning of physics and any other comment on the lesson. The students were unanimous in expressing their enthusiasm and appreciation of the teaching strategies that were used (refer to Chapter 7). Samples of the filled questionnaires of the students are given in Appendix 4.

Chapter 7

Findings

This study has enabled us to identify the type of classroom transactions occurring in our schools. We already mentioned that learning is enhanced when opportunities are provided in a variety of ways that meet the individual needs of the students. Our main concern in conducting this research study was to propose a different paradigm in the teaching-learning transaction in relation to the interrelationship between physics and mathematics. Physics students usually rote learn algorithms to solve numerical problems without having developed skills that will enable them demonstrate conceptual understanding (Mazur, 1996).

Teachers' Questionnaire

Data from the teachers' questionnaires reveal that there is basically no problem in the teaching-learning process and that teachers are fully aware of the problems that students encounter. According to the teachers, students are given due attention in the learning process and linkages between mathematics are made by the physics teacher. One of the responses is:

> "*students are aware that knowledge of algebra and calculus are important so that they are keen to keep in touch with their mathematics"* or another response:
> *"But I don't think they make good use of it. They have sometimes difficulties in linking the two subjects"*

When the teacher was asked to mention some of the problems that students encountered while studying physics at HSC level, the answer was "*they have problem with relating the mathematical part of the physics with the theory part"*. This answer is a stark contradiction of the above answer.

From the above statements, it is clear that teachers have based their presupposition on unfounded assumption.

When asked whether they test the mathematical knowledge during physics lessons, all the teachers acknowledged positively.
When asked whether they discuss the mathematical difficulties with the mathematics teacher, 75% acknowledged they do.
The following explanations are provided:

> "*try to find how they have learnt it in maths and explain afterwards using their maths knowledge. Most of the time making a recall of what have done in maths*"

> "*some notions are unclear or have not yet been taught to students hence they are given additional explanations such that their knowledge of the notions is clearer"*

Most of the answers provided by the teachers do not match our observations and feedback from students following the lessons conducted by the team of researchers. The triangulation of research tools provided sufficient reliability and validity to discard most of the teachers' comments.

Observation of physics classes

There was no evidence that teachers teaching physics concepts had recourse to core constructs. The teaching was the usual stereotyped practice whereby students were provided with information without involving them in in-depth reasoning processes. Though some of the teachers mentioned during the unstructured interviews that they enable students to link mathematics with physics, there was no such evidence during the observation phase. Such mismatch between teachers' assertion and their practice has been reported in other studies; for instance, Borg and Gatt (2004) confirmed that though teachers claimed that experiments are carried out in classrooms, it is not evident in practice.

Since, in general, the students have not been exposed to a variety of teaching strategies, they find the actual classroom transaction quite normal. Probably, the situation might have been different if the students were exposed to different strategies.

The mode of lesson delivery has not changed and it is the *chalk and talk* mode that still prevails in the classroom (Ramma, 2001). This type of classroom interaction encourages rote learning of concepts as it is the simplest way to get away with the prevailing system. Freire (1972) considers the teacher to be the "depositor" and the students to be the "depositories" (p. 46). It is unfortunate that most of the time students' pre-conception of facts are not challenged and as such, misconceptions still persist; conceptual misunderstanding, as a matter of fact, tend to cause students to advertently shy away from science.

The role and function of teachers are to enable learners to construct knowledge and they should not be seen to be part of the causes of failures of students. It is a well known fact that there are a number of factors affecting students' decision to opt for non science subjects (Ramma, 2001). The teaching strategies adopted by teachers should in principle not be responsible for such a problem. Unfortunately, there is ample evidence to certify that this is not the case. We firmly believe that if the classroom transaction between teachers and learners is based on an integrative approach, the situation will be different.

Students' evaluation of physics lessons

The student questionnaires (appendix 3) were submitted to all students present during the lesson.

It was very pleasing to find out that students have really appreciated the lessons, based on development of core constructs that were carried out by the research team.

When asked whether students liked the teaching, 100% agreed affirmatively. Some of the students' comments are:

> *The teaching was really interesting and along with the pictures and experiments ... has been explicit about his points. We enjoyed learning, that's what's needed, you must like what you learn. [C4, S4]*

> *Well it was the first time I sit about 2 hours to follow a class but you have a very good way of teaching that I have not feel bored. The teaching lesson was great. [C2, S3]*

This statement clearly shows that the teaching was appreciated. There is a need to point out that students' knowledge was constantly being challenged and they were lead to construct purposeful knowledge.

> *The students were allowed to express themselves freely and the teacher, based on these answers build up the topic explanation. I could find out that most of these explanations were linked with mathematical formulae which makes it much easier to learn. [C4, S5]*

This response clearly illustrates that this student has enjoyed the teaching and acknowledges that the mathematics link has permitted him/her to better understand the concept. Moreover, the student argues that continuous testing of prior knowledge provided opportunities for understanding.

The above response is in complete conjunction with what Puntambekar & Kolodner (2005) stipulate that the making of design decisions requires understanding of science issues. It should be noted that the role of the teacher is paramount to organising the classroom environment for successful learning. The teacher is therefore urged to create a conducive atmosphere that will enable students to tie concepts in different subject areas so that a complete and true picture of the concept under investigation is generated.

Another salient response by a student is:

> *Very good explained and taking into consideration the problems of the student to make him understand much better [C4, S17]*

The student acknowledges that the teaching did cater for individual differences of all the students. Researches (e.g., Agarkar, 2004; Goodell & Parker, 2003) have shown that teachers have to apply a variety of teaching strategies to meet the individual needs of learners so that they are able to bridge the gap between isolated concepts thus enabling them to make sense of physical phenomena in a logical way.

Teachers have a crucial role to play in enabling students to construct knowledge. Matthews (1994) rightly forecasts that:

> they [teachers] need to do more than just teach: they need either to develop local curricula, or to interpret national or provincial curricula for local use, they take part in school governance, and in policy-making that bears upon subjects taught in their school, and the levels to which subjects are taught to what students (p. 199).

It is worth noting that teaching is an art and that teachers have some kind of freedom to experiment new strategies with a view to identifying misconceptions that students have withheld. Once misconceptions have been identified and grouped in categories, teachers should now resort to dislodge them so that learning can take place. There is also a need to acknowledge that such a process is an effortful one but rewarding. After all, we all want our students to perform to the best of their abilities. The well designed strategies will enable the teachers to accumulate experience that "could spark a new explanation or hypothesis" (Valiela, 2001, p. 3). The open ended questions as well as the students centered approach enabled the students to probe further in their understanding of concepts. Some salient responses from the students clearly illustrate that they can be motivated if the teaching strategies make provision for that.

> *The questions were direct. Some were a bit complicated at first but after the explanation it was understandable.* [C 3, S10]
>
> *It's a new way of teaching for us.* [C3, S4]

The use of core constructs to teach physics was new to the students but once students are engaged in this new thinking process, they are able to appreciate the difference and thus enjoy the teaching. Lenton and Stevens (2002) maintain that:

> "Only by giving them [pupils] time and opportunity to estimate, calculate, interpret and discuss with each other and with the teacher, will they really gain a full understanding of the numerical and other mathematical concepts within science", (p. 145).

It was pleasing to note that the students were all the time involved in the thinking process that they find the teaching very conducive for conceptual development.

> *The class was not boring at all. There was a good interaction between the teacher and the students.* [C2, S 20]
>
> *By taking examples from our everyday life this helped us to have a better understanding.* [C4, S 12]
>
> *The fact that the examples are shown in another way as it is in books. The examples with moving diagrams help us to understand better the subject.* [C2, S 5]
>
> *It was amazing. It is very different from our normal class. During those two days I was not tired at all.*[C2, S 8]

The use of students centred teaching strategies, in conjunction with core constructs, in the development of logico-mathematical concepts has enabled us to lead students to adopt appropriate learning strategies necessary for the development of an effective "organising principle dictating their knowledge structure" (Kinchin, 2001, p. 92).

Chapter 8

Conclusion and Recommendation

Physics is considered as one of the most difficult subjects in the secondary school curriculum (for e.g. Raw, 1998). Many studies have been carried out to investigate into the problems students encounter while studying this subject (Johnson and Murphy, 1986; Branson et al., 1998; Watts and Pope, 1989) as well as factors influencing their achievement in physics (for e.g Akatugba and Wallance (1999); Blin-Stoyle (1993); Fisher (1990)). Though all these factors, in one way or the other, affect performance in physics, the fundamental assumption propounded by the research team is that the method of teaching adopted in the classroom can bring about a significant positive shift in attitude in the learners. Motivation to learn the subject is also enhanced.

One of the fundamental basis in physics learning lies in students' ability to think critically and to be able to engage in conceptual reasoning. For this to happen it is necessary that the classroom interaction between teacher and student, student and teacher, students and students enable the learners to develop and form conceptual framework.

Mathematics plays an important role for the development of conceptual understanding in physics; one who is able to apply mathematics in physics in a variety of perspectives would undoubtedly have developed core constructs.

With a view to enabling the enhancement of teaching and learning of physics at the Higher Certificate level, this study was designed with the following aims:
To:

(i) determine whether the teaching of physics is carried out comprehensively using appropriate mathematical concepts,

(ii) find out whether students encounter conceptual difficulty in learning physics,

(iii) investigate whether there is awareness of logico-mathematical concepts in physics from (a) the teachers', (b) the students' point of view.

(iv) Investigate whether there are difficulties in the teaching of physics (from the teachers' point of view),

(v) Find out whether students encounter difficulty in logic related to problem solving in physics.

- The questionnaire that was set to test for prior knowledge of the students clearly revealed that there was a lack of conceptual understanding; the mismatch between what was taught and what was understood came out clearly. This was further brought out during the observation phases.

- Classroom observations were carried out to gain an insight into the kind of teaching and learning transactions that were prevalent during the physics lessons. The lessons were mostly teacher centred, students were merely passive learners and their participation consisted mainly in answering closed questions set by the teacher. Opportunities were not given to students to make the link between mathematics and physics. The teaching tended to be stereotyped traditional mode (lecture type) and emphasis was laid on algorithm for solving problems. When the closed questions were set, our research team could identify misconceptions and inconsistencies in students' responses. Most of the time, these misconceptions were not probed into and were left untreated by the teacher. Further evidence of these misconceptions was obtained through the analysis of the students' questionnaires on prior knowledge.

- Our investigation led us to conclude that though teachers claimed to be aware of the importance of logico-mathematical concepts in the learning of physics, there was no evidence that their teaching strategies contained elements of core constructs.

- A thinking model paradigm has been constructed with a view to enabling the researchers to develop a conceptual framework of core constructs. In addition to that the Comprehensive Interactive Model (CIP) of Evaluation was used during the teaching phase to better evaluate and provide corrective measures during the lessons.

- After having identified the common core constructs following a meticulous analysis of the mathematics and physics syllabi at lower and upper secondary levels, lessons were developed and emphasis on the interrelationship between mathematics and physics concepts was clearly spelt out. During the lessons, students were led to continuously adopt a critical approach to learning. Since there were integration of ideas and concepts across the curriculum, the students were able to provide justification of different physical phenomena, making use of appropriate mathematical concepts. Such a perpetual level of interaction between the researchers and learners (multi-directional) and the use of core constructs undoubtedly motivated and raised the level of satisfaction of the students. This surfaced from the feedback questionnaires.

Teachers have a tremendous amount of knowledge and it is necessary that they be trained on how to use this knowledge in a holistic perspective so as to enable the learners to develop core constructs in physics. It is unfortunate that students become unmotivated during physics lessons as their interests are not sustained. Our observations of physics lessons led us to conclude that the teaching has not changed since the time when we were students ourselves. The findings confirm our assumption that the use of core constructs can bring about a paradigm shift in the teaching and learning. Moreover, the findings have enabled us to verify and confirm that cognitive acceleration of the learners may be achieved if the element of core construct is fully applied during the lessons.

Finally, the Mauritius Institute of Education will have to play a leading role in offering professional development courses based on core constructs.

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Please read carefully the statements and tick the most correct answer. There may be more than one correct answer. You may tick more than one box.

1. When we are measuring the length of an object, say a table, we are actually:

	A	determining the dimensions of the object
	B	making a comparison against a standard length
	C	calculating the thickness of the object
	D	calculating the volume of the wood used
	E	determining the type of structure of the wood

2. Which graph shows that motion has stopped?

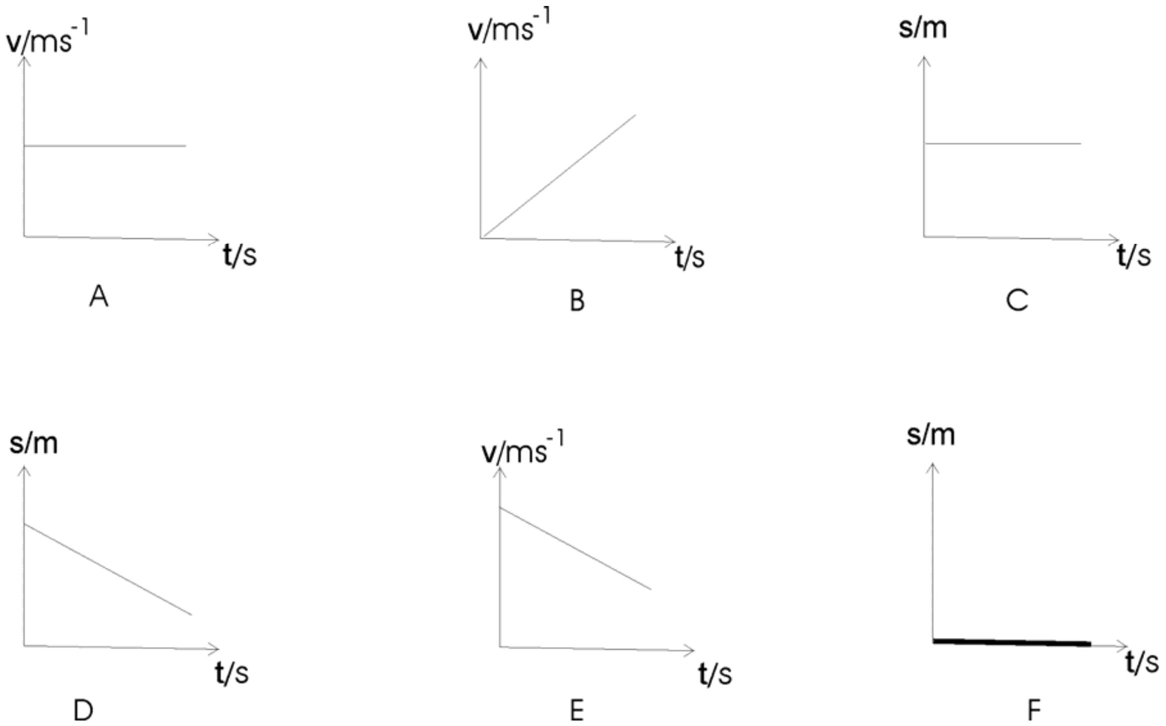

Figure 1

3. When an object is in a state of free fall (neglecting air resistance), the following statement is true:

	A	The velocity remains constant
	B	The velocity changes
	C	The velocity is zero
	D	The acceleration changes
	E	The acceleration remains constant
	F	The acceleration is zero

4. Figure 2 shows a displacement – time graph of an object.

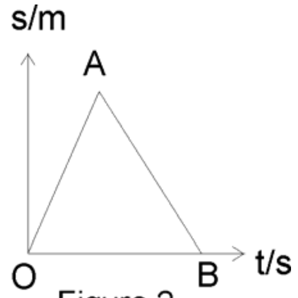

Figure 2

Lines segments OA and AB in figure 2 represent:

	A	motion with uniform velocity
	B	motion with acceleration
	C	motion with increasing speed
	D	motion with non-uniform acceleration
	E	motion with decreasing speed
	F	linear motion

5. **Line segments OA and AB in figure 2 represent:**

	A	motion uphill and downhill
	B	circular motion
	C	perpetual motion
	D	motion in different directions
	E	motion in one direction only
	F	no motion at all

6. **A stone is tied to a string and rotated in a circle (figure 3). When the stone is in position 'A', the string breaks. In which direction will the stone move?**

	A	1
	B	2
	C	3
	D	4
	E	5

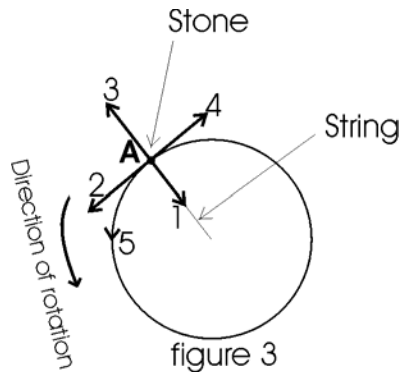

figure 3

7. **An object attains terminal velocity when:**

	A	it decelerates instantaneously
	B	it accelerates non-uniformly
	C	the resultant forces acting on the object is zero
	D	the resultant forces acting on the object is not zero
	E	it is falling with constant velocity
	F	it has reached a constant height

8. An equal mass of ice and water have different densities because:

	A	they do not have the same weight
	B	they are not made up of the same elements
	C	they occupy different volumes
	D	they are of different colours
	E	they have different structures
	F	they have different properties

9. The relationship between force and acceleration is given by F = m x a, **F represents:**

	A	a single force
	B	the net force
	C	an opposing force
	D	a revolving force
	E	the resultant force
	F	the vector sum of various forces

10. The following information can be gathered from the equation F=ma

	A	mass is a vector quantity
	B	acceleration acts along the direction of the applied force
	C	the product of a scalar and vector quantity results in a vector quantity
	D	the base SI units on each side of the equation are not equivalent
	E	the equation is homogeneous
	F	the equation is non-linear

11. From Hooke's law, where F is the applied force, k is the spring constant, e is the extension. The spring constant 'k' has:

	A	the same value for all springs
	B	different value for all springs
	C	the same value for the same type of springs
	D	the same value for different type of springs
	E	the same value for the same length of similar type of springs
	F	the same value for any length of similar type of springs

12. Frictional force is a force that opposes motion. Figure 4 shows a car moving from left to right. In which direction does frictional force acts?

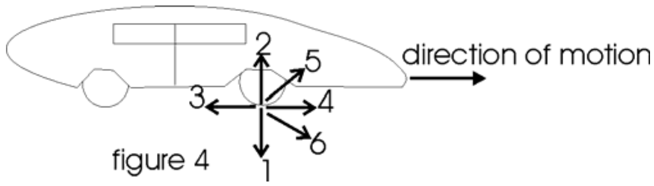

figure 4

	A	1
	B	2
	C	3
	D	4
	E	5
	F	6

13. When a ray of white light enters perpendicularly a dense medium from a less dense one, the following statement is true:

	A	the ray is refracted towards the normal
	B	the ray is refracted away from the normal
	C	the ray bounces away from the less dense medium
	D	the ray passes through without being deviated
	E	the ray is totally internally reflected in the dense medium
	F	the ray breaks into its seven components

14. The figure below shows a balloon maintained at the bottom of a 10 m pond. When the string is cut, the following statement is true:

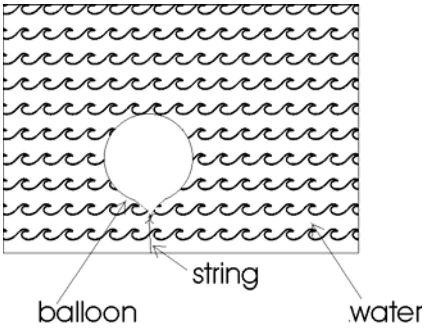

figure 5

	A	the balloon remains undisturbed and its size decreases
	B	the balloon remains undisturbed and its size increases
	C	the balloon will stay at the same position unaffected
	D	the balloon will rise to the surface and may burst
	E	the balloon will move in a horizontal direction

15. A magnetic field is represented by a directed line segment. The length and the arrow indicate the following:

	A	strength and uniformity of the field
	B	density and direction of the field
	C	magnitude and direction of the field
	D	magnitude and strength of the field
	E	strength and orientation of the field

16. **Air is not used as an insulator provided it is trapped in small compartments, as in wool and fibreglass because:**

	A	air is a good conductor of heat unless trapped
	B	air is a bad conductor of heat when trapped
	C	air will transfer heat by convection unless trapped
	D	air molecules will vibrate and transfer the heat unless trapped
	E	the atoms constituting the air molecules is a good conductor of heat unless trapped
	F	air form bonds with each other when trapped

17. **We can reduce the possibility of breaking a glass jar when pouring boiling water into it by placing a metal spoon into the jar. This is because:**

	A	the spoon prevents heat to conduct away
	B	the spoon is a bad conductor of heat
	C	the spoon will conduct heat away from the glass
	D	the spoon will prevent heat from entering the glass
	E	the spoon will transfer the heat slowly to the glass
	F	the spoon will transfer the heat quickly to the glass

18. **A current can flow between two points in a circuit because:**

	A	the circuit is closed
	B	the circuit is open
	C	there is a cell in the closed circuit
	D	there is a potential between the two points
	E	there is a potential difference between the two points
	F	there is no resistance at all in the circuit

19. In a circuit, cells are not connected in parallel because:

	A	the emf will be too low
	B	the emf will be too high
	C	the resistance will be too low
	D	the cells will leak
	E	the current will still flow even if the switch is open
	F	the current will not flow even if the switch is closed

20. Which meter from figure 6 is not correctly connected?

	A	1
	B	2
	C	3
	D	4
	E	5
	F	6

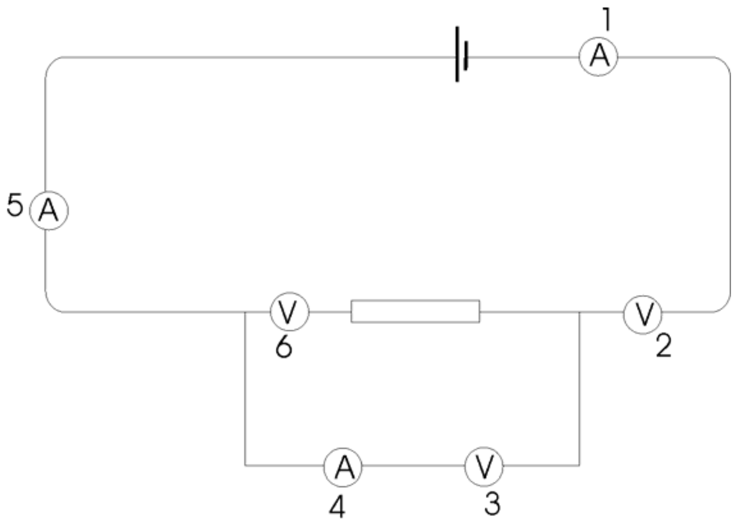

figure 6

Thanking you for your cooperation.

Research team, Mauritius Institute of Education

Key concepts	Yes	No	Remarks
Aims of the lesson are clearly stated			
Learning outcomes of the lesson are clearly stated			
There is evidence of preliminary Planning (lesson plan, resources, demonstration set, activities, etc)			
Prior knowledge tested 1. Question are set to test prior knowledge of previous lesson			
2. Question are set to test prior knowledge in everyday life situations			
3. Question are set to individual students at the right time and adequate prompts are provided			
4. Wait time is adequate			
Development of the lesson 1. A good introduction of topic is provided			
2. The development of the lesson is systematic (step-by step with prompts) and well organized			
3. Students are engaged into critical thinking (a) The questions that are set are in accordance with the following criteria			

Observation schedule

(i) K***nowledge*** (notion, recall of information, facts….)			
(ii) ***Comprehension*** (translation of material from one form to another by interpreting material and estimating)			
(iii) ***Application*** (learned material is transformed into anther form, including rules, methods, concepts and principles)			
(iv) ***Analysis*** (includes analysis of parts of material, analysis of relationships between parts)			
(v) ***Synthesis*** (ability to add parts together to form a new whole)			
(vi) ***Evaluation*** (ability to judge the value of the material)			
(b) The teacher provides ample time for the students to answer questions (at least 10 seconds/questions)			
(c) The teacher simplify the task so that students can manage			

on his/her own			
(d) Students are encouraged to ask questions			
4. The explanation is clear and understandable			
5. Ample examples are set to help students analyse the various concepts developed during the lesson			
6. Questions set by the teacher provoke critical thinking (the guidance is systematic and misconceptions are uprooted)			
7. Teacher always refer to mathematics knowledge during the physics lesson			
8. Teacher shows relationship between mathematics and physics concepts during lesson development (in physics)			
9. A brief recall of the maths concepts are exposed and the link to physics shown			
10. Students are encouraged to link acquired maths knowledge to physics			
11. A good conclusion to the lesson is provided and students are encouraged to reflect on the lesson			

Additional remarks:

Questionnaire

You are kindly requested to answer the following questions and please note that your responses will be treated confidentially

1. Do you like the way the teaching was done? | Yes ✓ | No |

Please clarify

We learned how to use our knowledge of other subjects in one i.e. how to use our maths knowledge in Physics. (Linking of knowledge of different subjects)

2. Do you think that the questions that were set were clear? | Yes ✓ | No |

Please explain

The questions were clear since the situation was given to us very clearly and some questions were also asked in Mathematical terms

3. Do you think that the questions set during the development of the lesson allowed you to better understand the concepts? | Yes ✓ | No |

Explain

The lecturer himself lead us towards the answer

4. Were the questions challenging? | Yes ✓ | No |

5. Do you appreciate the link between mathematics and physics during the lesson development? | Yes ✓ | No |

6. Do you think that you will better understand physics when the link between mathematics and physics is made? | Yes ✓ | No |

Explain

The logic used is the same

Any addional comment/suggestion (you may write overleaf) This was a very good way of teaching. it was not only limited to

Thanking you, MIE Research Team 01 October 2003

Questionnaire

You are kindly requested to answer the following questions and please note that your responses will be treated confidentially

1. Do you like the way the teaching was done? | Yes ✓ | No |

Please clarify

The way the teaching was done allow me to understand better the concepts of S.H.M. The way of teaching was clear and very much understanding.

2. Do you think that the questions that were set were clear? | Yes ✓ | No |

Please explain

Yes, for every question the answers were clear. The answers help me in understanding better the lesson.

3. Do you think that the questions set during the development of the lesson allowed you to better understand tl concepts? | Yes ✓ | No |

Explain

The questions were linked directly with the lesson, ~~so as to develop~~ The concepts became much clear.

4. Were the questions challenging? | Yes ✓ | No |

5. Do you appreciate the link between mathematics and physics during the lesson development? | Yes ✓ | No |

6. Do you think that you will better understand physics when the link between mathematics and physics is made? | Yes ✓ | No |

Explain

~~Yes,~~ It made me realise that without mathematics, physi cannot be understood and vice versa.

Any addional comment/suggestion (you may write overleaf) I appreciate this way of teaching very much

Thanking you, MIE Research Team 01 October 2003 Thank You Sir.

Questionnaire

You are kindly requested to answer the following questions and please note that your responses will be treated confidentially

1. Do you like the way the teaching was done? | Yes ✓ | No |

Please clarify

The teacher was of a very high quality and cooperated very well with the students. Everyone was given the chance to voice out their answers

2. Do you think that the questions that were set were clear? | Yes ✓ | No |

Please explain

It was a bit difficult but when explained it become easy to understand

3. Do you think that the questions set during the development of the lesson allowed you to better understand t concepts? | Yes ✓ | No |

Explain

We need to bring logic and maths in physics which was never told by our respective teachers.

4. Were the questions challenging? | Yes ✓ | No |

5. Do you appreciate the link between mathematics and physics during the lesson development? | Yes ✓ | No |

6. Do you think that you will better understand physics when the link between mathematics and physics is made? | Yes ✓ | No |

Explain

Yes this will improve our knowledge in both fields especially physics

Any addional comment/suggestion (you may write overleaf) Thank you for your limited tim

Thanking you, MIE Research Team 01 October 2003

Questionnaire

You are kindly requested to answer the following questions and please note that your responses will be treated confidentially

1. Do you like the way the teaching was done? Yes ✓ | No

Please clarify

The teacher was very friendly and he took care of clarifying the issues that were a problem for us. He also took care of going around and helping those that were a little confused.

2. Do you think that the questions that were set were clear? Yes ✓ | No

Please explain

They were very direct questions, demanding mainly mathematics knowledge.

3. Do you think that the questions set during the development of the lesson allowed you to better understand the concepts? Yes ✓ | No

Explain

The questions were placed at check points, to test without marking, the lessons.

4. Were the questions challenging? Yes ✓ | No

5. Do you appreciate the link between mathematics and physics during the lesson development? Yes | No ✓

6. Do you think that you will better understand physics when the link between mathematics and physics is made? Yes | No ✓

Explain

Mathematics and physics are not poles apart but they are identical. Physics should be explained theoritically with words and definitions.

Any addional comment/suggestion (you may write overleaf) Thank you for your generous time

Thanking you, MIE Research Team 01 October 2003

Questionnaire

You are kindly requested to answer the following questions and please note that your responses will be treated confidentially

1. Do you like the way the teaching was done?

Yes	No
✓	

Please clarify

The students were allowed to express themselves freely and the teacher, based on these answers build up the topic explanation. I could found out that most of these explanations were linked with mathematical formulaes which makes it much easier to learn.

2. Do you think that the questions that were set were clear?

Yes	No
✓	

Please explain

The explanation started in a very simple manner and then a more deep teaching was done

3. Do you think that the questions set during the development of the lesson allowed you to better understand the concepts?

Yes	No
✓	

Explain

Different types of examples and questions were set which increase the knowledge field, then based on simple explanations made at the start, we were able to understand much better

4. Were the questions challenging?

Yes	No
	✓

5. Do you appreciate the link between mathematics and physics during the lesson development?

Yes	No
✓	

6. Do you think that you will better understand physics when the link between mathematics and physics is made?

Yes	No
✓	

Explain

Since mathematics are very easy, interesting and challenging to use, it makes physics much interesting to learn

Any addional comment/suggestion (you may write overleaf)

Thanking you, MIE Research Team 01 October 2003

Questionnaire

You are kindly requested to answer the following questions and please note that your responses will be treated confidentially

1. Do you like the way the teaching was done? | Yes ✓ | No |

Please clarify

The teaching was really interesting and along with the pictures and experiments shown on the projector, Dr Y Ramma has been explicit about his points. We enjoyed while learning, that's what's needed, you must like what you learn

2. Do you think that the questions that were set were clear? | Yes ✓ | No |

Please explain

The questions that were set were clear but still from time to time, there are lots more questions that come and that should be introduced in the teaching session.

3. Do you think that the questions set during the development of the lesson allowed you to better understand th concepts? | Yes ✓ | No |

Explain

As

4. Were the questions challenging? | Yes ✓ | No |

To quite an extent! Not very very much!

5. Do you appreciate the link between mathematics and physics during the lesson development? | Yes ✓ | No |

6. Do you think that you will better understand physics when the link between mathematics and physics is made? | Yes ✓ | No |

Explain

It's better to use what you already know to get a better know-how.

Any addional comment/suggestion (you may write overleaf) We'll really appreciate if you come again. Thanks

Thanking you, MIE Research Team 01 October 2003

Questionnaire

You are kindly requested to answer the following questions and please note that your responses will be treated confidentially

1. Do you like the way the teaching was done? Yes ✓ | No

Please clarify

The teaching was very good and interesting. It was of a good benefit to us.

2. Do you think that the questions that were set were clear? Yes ✓ | No

Please explain

They were clear enough for us to understand the basis more easily

3. Do you think that the questions set during the development of the lesson allowed you to better understand the concepts? Yes ✓ | No

Explain

Through discussion of the answers, it was more easy to understand the concepts

4. Were the questions challenging? Yes ✓ | No

5. Do you appreciate the link between mathematics and physics during the lesson development? Yes ✓ | No

6. Do you think that you will better understand physics when the link between mathematics and physics is made? Yes ✓ | No

Explain

During the lesson we have seen that it becomes very difficult to understand the concepts if we are not good enough in Mathematics

Any addional comment/suggestion (you may write overleaf)

Such sessions should be organised more frequently as it is a good way to develop our thinking and logical skills

Thanking you, MIE Research Team 01 October 2003

Questionnaire

You are kindly requested to answer the following questions and please note that your responses will be treated confidentially

1. Do you like the way the teaching was done? | Yes ✓ | No |

Please clarify

It was very different from how our class teacher usually teached. It was much more interesting and the atmosphere was much more relaxed too.

2. Do you think that the questions that were set were clear? | Yes ✓ | No |

Please explain

I can say so! It was in simple English, so I don't think anyone had had any problem understanding them.

3. Do you think that the questions set during the development of the lesson allowed you to better understand the concepts? | Yes ✓ | No |

Explain

~~Yes~~ It helped to understand the concepts better as the questions tend to make us think deeper into the subject.

4. Were the questions challenging? | Yes ✓ | No |

5. Do you appreciate the link between mathematics and physics during the lesson development? | Yes ✓ | No |

6. Do you think that you will better understand physics when the link between mathematics and physics is made? | Yes ✓ | No |

Explain

I've just learnt that physics and mathematics are linked in many ways. To learn physics, I think someone must know maths well so that he could appreciate physics.

Any addional comment/suggestion (you may write overleaf)... Thank you! Hope you come back soon.

Thanking you, MIE Research Team 01 October 2003